2-D DIGITAL WAVEGUIDE MESH TOPOLOGIES IN ROOM ACOUSTICS MODELLING

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ABSTRACT

Digital waveguide mesh models have provided an accurate and efficient method of modelling the properties of many resonant structures, including acoustic spaces. 2-D rectilinear and triangular mesh structures have been used extensively in the past to model plates and membranes and are presented here as potential analogues to 2-D acoustic spaces. Impulse response measurements are taken and comparisons are made regarding the spectral content and the associated properties when compared with standard room acoustic parameters. Enhanced mesh structures are examined using frequency warping techniques and high-resolution sampling rates. The 2-D triangular mesh is shown to be considerably superior to the rectilinear mesh in terms of the measurements taken, with a further significant improvement being made by using the same mesh oversampled to a much higher resolution to improve the bandwidth of the measured impulse responses.

1. INTRODUCTION

All sounds have associated with them an environmental context related to the acoustic space within which they are heard. Composers have long used and manipulated these properties of sounds in space as a fundamental part of their music. Many methods have been used to simulate the acoustics of an enclosed space, and digital waveguide mesh models have provided an accurate and efficient method of modelling this physically complex system [1], [2], [3], [4]. However, due to the high sampling rate that is required to model even the smallest and simplest acoustically interesting space, a 3-D mesh suitable for full audio bandwidth processing is often impractical to implement, resulting in prohibitively high computation times. A partial compromise is sought by accurately modelling an acoustic space in the two dimensional plane only, in order to ascertain whether or not a musically useful environmental context can be successfully synthesized [5].

This paper examines some of the properties of two such 2-D waveguide mesh structures when used to model a simple rectangular 2-D representation of an enclosed space. Both the rectilinear and triangular mesh structures have been used extensively in the past to model plates and membranes [6], [7] and are presented here as potential analogues to 2-D acoustic spaces. Impulse response measurements are taken and comparisons are made regarding the spectral content and the associated properties when compared with standard room acoustic parameters. Suggestions are also made as to how these 2-D models could be enhanced so as to improve the resultant acoustic environmental context.

2. 2-D DIGITAL WAVEGUIDE MESH STRUCTURES

A waveguide is any medium in which wave motion can be characterised by the one-dimensional wave equation. In the lossless case, all solutions can be expressed in terms of left-going and right-going travelling waves and can be simulated using a bi-directional digital delay line. A digital waveguide model is obtained by sampling, both in space and time, the one-directional travelling waves occuring in a system of ideal lossless waveguides [8]. The sampling points in this case are called scattering junctions, and are connected by bi-directional unit-delay digital waveguides [9]. Figure 1 shows the general case of a scattering junction J with N neighbours, $i = 1,2,\ldots N$.

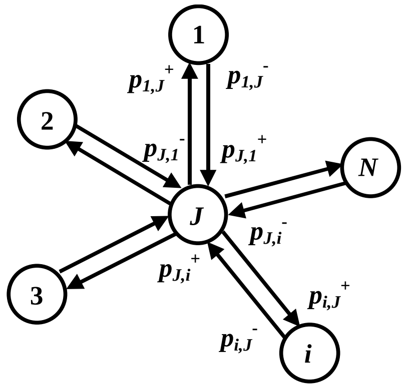

Figure 1. *A general scattering junction J with N connected waveguides for i = 1,2,…,N.*

The sound pressure in a waveguide is represented by p_i, the volume velocity by v_i and the impedance of the waveguide by Z_i. The input to a waveguide is termed p_i^+ and the output p_i^-. The signal $p_{i,J}^+$ therefore represents the incoming signal to junction i along the waveguide from the opposite junction J. Similarly, the signal $p_{i,J}^-$ represents the outgoing signal from junction i along the waveguide to the opposite junction J. The volume velocity v_i is equal to pressure, p_i, divided by impedance, Z_i. The delay elements are bi-directional and so the sound pressure is defined as the sum of its input and output:

$$p_i = p_i^+ + p_i^- \qquad (1)$$

At a lossless scattering junction with N connected waveguides the following conditions must hold:

1. $\sum_{i=1}^{N} v_i^{+} = \sum_{i=1}^{N} v_i^{-}$
2. $p_1 = p_2 ... = p_i = ... = p_N$

Using these conditions the sound pressure at a scattering junction can be expressed as:

$$p_J = 2\frac{\sum_{i}^{N}\frac{p_i^{+}}{Z_i}}{\sum_{i}^{N}\frac{1}{Z_i}} \qquad (2)$$

As the waveguides are equivalent to bi-directional unit-delay lines, the input to a scattering junction is equal to the output from a neighbouring junction into the connecting waveguide at the previous time step. This can be expressed as:

$$p_{J,i}^{+} = z^{-1} p_{i,J}^{-} \qquad (3)$$

To model the propagation of a wave on the horizontal plane within an enclosed space, 2-D rectilinear and triangular mesh structures are constructed using unit delay waveguides and lossless scattering junctions with N = 4 and N = 6 in Equation 2 respectively. A signal representing acoustic pressure introduced to a waveguide will propagate in either direction along the bi-directional delay lines until it comes to a junction. The signal then scatters according to the relative impedances of the connected waveguides. In the current model all impedances are set to be equal. Simple absorption can be modelled at a boundary where a general scattering junction is replaced with the equivalent n-port junction (where n=1,2,…5), according to the room/boundary geometry that the mesh model has to fit. Figure 2 shows an example of the two mesh structures used to model a simple 2-D enclosed space using both the rectilinear and triangular mesh topologies.

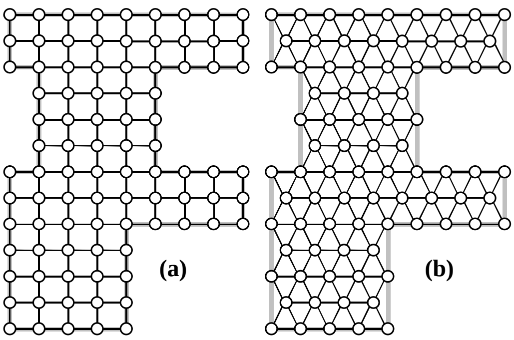

Figure 2. *A simple 2-D enclosed space and the resultant mesh structures used. (a) rectilinear mesh topology; (b) triangular mesh topology.*

3. THE *WAVEVERB* DIGITAL WAVEGUIDE MESH REVERBERATION SYSTEM

The *WaveVerb* System has been developed to allow Room Impulse Responses (RIRs) to be generated from a 2-D representation of an enclosed space using both rectilinear and triangular mesh topologies, with an emphasis on its use as a high level creative tool for the computer musician [10]. Figure 3 shows the user interface that allows the resultant wave propagation to be monitored on a graphical display with a number of different views and realisations. This provides visual feedback on the behaviour of the propagating wave and the properties of the room itself at low mesh sampling rates. This module forms the basis of the *WaveVerb* system and allows the user to make RIR measurements for various source-listener positions. Control is given over variables such as room width, length and mesh density. In this way the user does not require any in depth knowledge of room acoustics in order to generate a RIR for a reverberant effect - the parameters the user has control over correlate readily to real world variables such as room size, source location and object position. Wave phenomena such as diffraction, reflections and interference are natural consequences of the model. A command line implementation of *WaveVerb* is also used without the option for feedback or user interaction to facilitate full audio bandwidth processing at much higher mesh sampling rates.

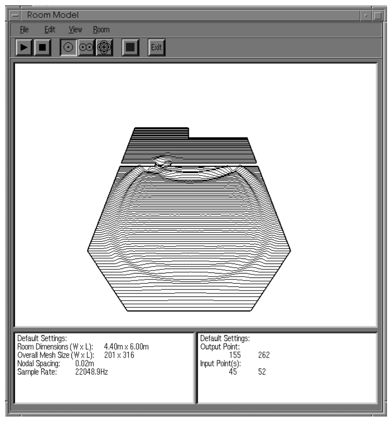

Figure 3. *The WaveVerb System: The Wave Propagation Interface*

4. MESH LIMITATIONS AND DISPERSION ERROR

There are two main problems that tend to limit the ability of waveguide mesh structures to propagate high frequencies successfully. The sampling rate of the mesh (in Hz), which is determined by the spacing of the scattering junctions and the mesh topology, is given by:

$$f_{update} = \frac{c\sqrt{2}}{d} \qquad (4)$$

Where c is the speed of sound (in ms^{-1}), and d is the distance between scattering junctions (in m). The higher the frequency that is required to be propagated by the structure, the denser the mesh has to be in terms of the number of scattering junctions used, and the more time it takes to calculate and obtain a satisfactory RIR. This problem can be solved effectively by increasing the computational power available for the mesh algorithm or by using a parallel implementation [11].

The effective sampling rate, and so the wave of highest frequency that can be propagated successfully by the mesh, is well below the value given by f_{update}. An inherent problem with lattice type structures such as the digital waveguide mesh is that they exhibit frequency and angular dependent dispersion, whereas in the ideal case all wave frequencies travel at the same speed in every direction. This property is dependent upon the mesh topology as well as the mesh density [12]. The dispersion error present in the basic rectilinear mesh can be improved upon by using a triangular mesh constructed using 6-port scattering junctions. This topology is the most efficient decomposition of the 2-D plane in terms of the consistency and minimisation of the dispersion error such that it is almost totally independent of the direction of propagation [13]. The dispersion error is therefore only a function of frequency, rather than frequency and direction of wave propagation. The availability of increased computing speed further minimises dispersion error that is dependent only on wave frequency by using a mesh of a density sufficient enough to yield a result valid for the bandwidth required.

5. RESULTS

A number of RIR measurements for both triangular and rectilinear mesh topologies were made for a 2-D representation of a rectangular room, 7.0m long and 6.0m wide, with a fixed source, four different output points, and varied boundary absorption conditions. To generate a satisfactory audio rate RIR, a mesh sampling rate of at least 44.1kHz is required. Using Equation 4, this can be obtained assuming $c = 343\ ms^{-1}$ and $d = 0.011m$ giving f_{update} = 44098Hz. A 2-second Room Impulse Response (RIR) measurement was made in each case.

5.1. Observation of Wavefronts and Dispersion Error

Figure 4 shows a plan view of the wavefronts on both meshes 14 time steps after each mesh has been excited with a Gaussian impulse applied over 6 time steps. In Figure 4(a) the circular wavefront is clearly non-uniform, with "fuzzy" edges on the top, bottom and sides. This is due to the direction dependent dispersion error present on the rectilinear mesh. The wavefront along the diagonals to the mesh (if it were to be super-imposed on top of this image) are actually quite sharp and defined. It can be shown mathematically that there is no dispersion on the rectilinear mesh along the diagonals to the mesh coordinate system. However the top, bottom and sides of the wavefront are not so well defined due to the dispersion error being maximum in these directions. The wavefront on the triangular mesh, however, is much closer to being a uniform circle.

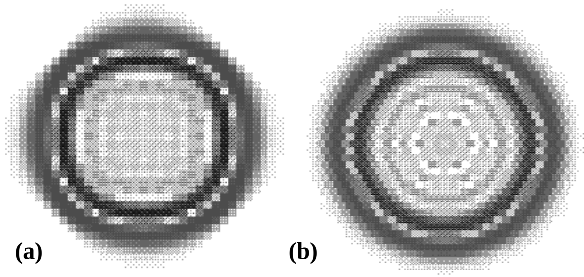

Figure 4. *Wavefronts on, (a) rectilinear mesh, and (b) triangular mesh, 14 time steps after an applied Gaussian impulse.*

5.2. Spectral Analysis

Figure 5 shows a spectral analysis for the output point furthest away from the input, with uniformly low absorption conditions across all boundaries.

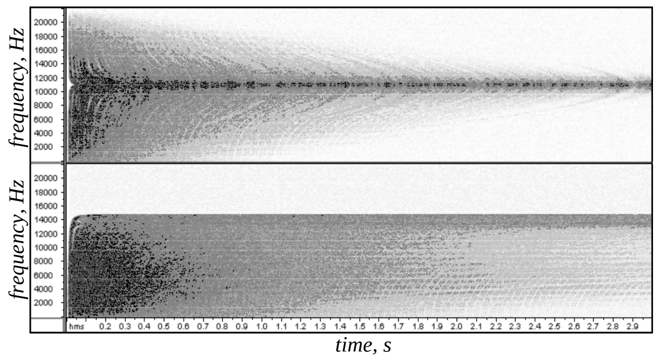

Figure 5. *Time varying spectral analysis of RIR from a single output point for, (top) rectilinear mesh, and (bottom) triangular mesh.*

Note that the triangular mesh exhibits a well defined cut-off point being the natural upper limit of the mesh above which no frequency can be successfully propagated. This cut-off is related to the mesh sampling rate, being equal to $f_{update}/3$, with f_{update} = 44.1kHz. In the case of the rectilinear mesh there is a well defined resonant peak present in all frequency response measurements at $f_{update}/4$, and it can be noted that the frequency response above this peak reflects that below it. The reason for this "mirroring" is due to the fact that when a Dirac impulse is applied as an input to the rectilinear mesh, every other sample value is equal to zero in the resultant RIR. This is because the path length of every route between two arbitrary junctions is either an odd or even number of waveguide elements, each being a unit delay. In the rectilinear mesh there does not exist a pair of junctions which can be reached by both an odd and even number of unit waveguide elements. Conversely in the triangular waveguide mesh, there exists both an odd and even length path between any two junctions. The implication of this property being that the aliasing present in the rectilinear mesh gives a frequency response only valid up to $f_{update}/4$ rather than $f_{update}/3$ in the case of the triangular mesh (ignoring any additional frequency response limitations due to dispersion error).

5.3. Reverberation Time Measurements

It can been seen from Figure 6 that the Reverberation Time (RT_{60}) measurements for both topologies are generally consistent across all of the output points. This agrees with the principle that the reverberant sound present in a room should be diffuse – that is, the reverberant sound visits all parts of the room with equal probability.

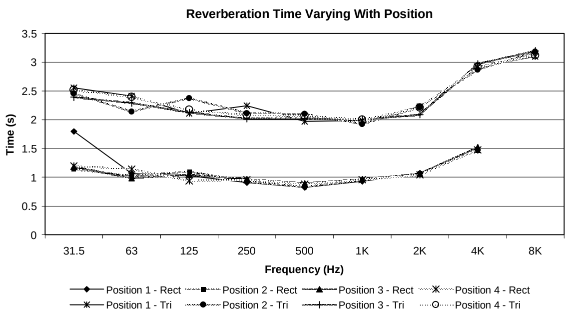

Figure 6. *Reverberation Time Measurements varying with output point position in the modelled room, for both the triangular mesh (upper set of measurements) and the rectilinear mesh (lower set of measurements).*

Figure 7 shows the spatially averaged RT_{60} measurements varying with total room absorption, rather than those particular to a given output point. This being a valid assumption due to the property evidenced in Figure 6. Setting the absorption level equally at all four walls to an appropriately low level produced the set of results labelled as “Low Absorption”. Similarly, setting the absorption level equally at all four walls to an appropriately high level produced the set of results labelled as “High Absorption 1”. The final set of results, labelled as “High Absorption 2” have been generated using a different absorption value at each wall.

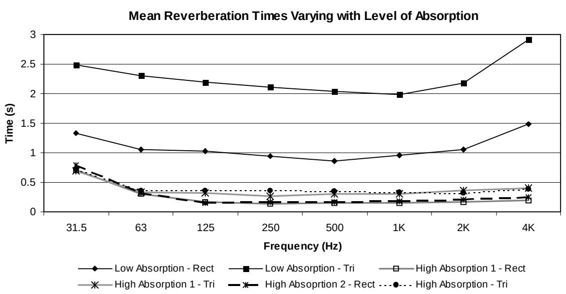

Figure 7. *Spatially Averaged Reverberation Time Measurements, varying with level of absorption in the modelled room.*

Note firstly that as the absorption increases the RT_{60} values fall, as would be expected with a real room. It is also evident that the RT_{60} values for the triangular mesh are approximately double those of the rectilinear mesh. Given that the RIRs from the rectilinear mesh have every other sample value equal to zero it is clear that there is a much lower average signal strength and hence less reverberant energy present. RIRs from the triangular mesh have double the number of non-zero samples and hence the RT_{60} measurements are correspondingly longer – approximately twice as long as those from the rectilinear mesh.

6. ENHANCED MESH STRUCTURES

It seems clear from the previous results that it is possible to generate an RIR useful for generating a musically useful acoustic environmental context. The triangular mesh has a considerably lower noise floor compared to the rectilinear mesh although with higher levels of absorption it is possible to detect changes due to the position of the output point relative to the input. This again ties in with real world expectations as it becomes more difficult to accurately place a sound source in a highly reverberant field. Sounds or notes with a percussive transient attack result in a slight, yet noticeable, high frequency resonance. This is in agreement with the observed physical behaviour of both mesh structures in that they do not respond well to high amplitude impulsive signals as this introduces high frequency distortion. However, the best acoustic effect is demonstrated using RIR measurements obtained from the triangular mesh with absorption levels set to be different for each wall, giving a more complex and random soundfield in the room. The resulting sounds are more full and natural and do not conflict with how the modelled space should be perceived in terms of its geometrical features. These results are further enhanced when a stereo RIR is used. However, there is still evidence of high frequency resonance.

Clearly a full 3-D mesh structure based on a triangular topology – or a dodecahedral topology in 3-D terms - would offer a significantly improved and more realistic environmental context, with a full complement of modal frequencies present, but at the expense of considerably higher computational overheads. A useful next step is to examine potential strategies for improving the quality of the results produced by the 2-D triangular mesh, to see if they can offer a partial solution.

6.1. Frequency Warping

It has been shown that the dispersion error of the triangular mesh - as it is effectively consistent across all directions of propagation - can be reduced by using frequency warping techniques [14] and [15]. This involves post processing the RIR generated by the mesh using a warped FIR filter that frequency shifts the signal, hence reducing the dispersion error. The FIR filter is implemented using a first-order allpass transfer function to replace each unit delay element. The allpass transfer function is of the form:

$$A(z) = \frac{z^{-1} + \lambda}{1 + \lambda z^{-1}} \qquad (5)$$

Where λ determines the extent of the frequency warping, being uniform for each allpass filter used in the FIR filter construction. The coefficients of the FIR filter are determined by the sample values of the RIR that is to be warped. An optimal warping factor for the range of frequencies from 0Hz to $f_{update}/4$ Hz, of λ = -0.109540 is used [15].

The warped triangular mesh variation of the 2-D rectangular room model analysed in Section 5 is implemented by initially dewarping the input signal by a factor of $-\lambda$, and applying the result to a resampled mesh with $d_{resampled} = D.d$, where D is defined by:

$$D = \frac{1-\lambda}{1+\lambda} \qquad (6)$$

Being the phase delay at low frequencies caused by the warping operation [15]. Finally, the output RIR is warped using a factor of λ.

6.2. High Resolution Mesh Structures

The limitations of dispersion error and sampling rate relate to the bandwidth over which the RIR can be said to be valid, which is generally accepted as being up to $f_{update}/4$ (although both the warped and non-warped triangular mesh structures improve on this for the same given sampling rate). Therefore a brute force solution to the problem is to oversample the mesh sufficiently so that the resultant RIR is valid over the required frequency range, being in this case, 0-22050Hz. The computational overhead of implementing this solution being still considerably less than a full 3-D solution, due to the total number of scattering junctions involved.

The triangular mesh rectangular room structure was resampled using d = 0.0055, giving f_{update} = 88196Hz, approximately equal to 2 x 44.1kHz.

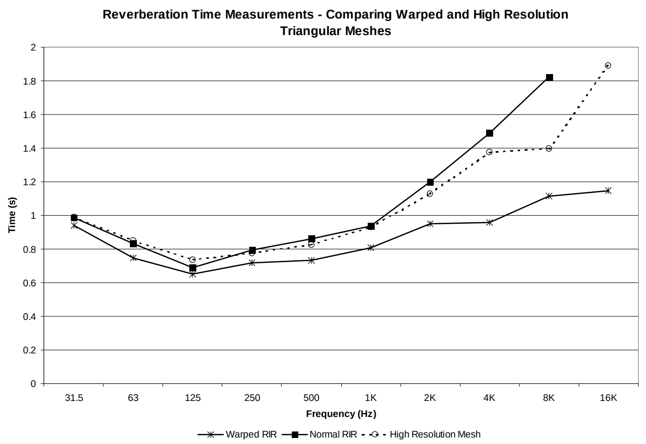

Figure 8. *Reverberation Time measurements for the triangular, high-resolution triangular, and warped triangular mesh structures.*

Figure 8 shows the RT_{60} measurements for a single output point in the modelled room for both of these enhanced mesh structures when compared with the standard triangular mesh. Note that there is a close correlation in the low frequency region between all three mesh implementations. This is as expected due to the high sampling rates used resulting in a highly accurate model at low frequencies [16]. However there is a divergence at the higher frequency bandwidths. This is most likely due to errors in the RT_{60} value calculation as both the triangular mesh and the warped triangular mesh still exhibit a distinct cut-off point. Therefore the only RIR that has a valid measurement across the whole bandwidth is the high-resolution mesh. Further, despite the fact that the warped mesh produces a more linear RT_{60} result varying with frequency, when convolved with anechoic audio, the RIR measured from the high-resolution mesh sounds far superior, with clear high frequency definition and a minimum trace of the high frequency resonance mentioned previously.

Note that audio examples and further details regarding these mesh models are available from the WWW URL address:

`http://www-users.york.ac.uk/~dtm3/`

7. Further Work

A number of further possibilities exist in developing these 2-D waveguide mesh models for room acoustics modelling. Future work will include the development of layered 2-D mesh structures to improve the modal density but without the overhead of a full 3-D model, and an investigation into high-resolution mesh structures for early reflection modelling. This would use a high-resolution mesh to model the early part of the RIR and a more generic method to model the late reverberant sound in a more computationally efficient manner.

8. Conclusions

The properties of a number of 2-D full audio bandwidth waveguide mesh structures have been examined and compared when used to model a simple rectangular 2-D representation of an enclosed space. The wavefronts present on the triangular and rectilinear mesh demonstrate the effect of dispersion error that is an inherent property of these models, and help to show that it is minimised for the triangular mesh topology. Spectral analyses of RIRs measured from both the rectilinear and triangular meshes show the improved high frequency response of the latter, with particular regard to the aliasing properties of the rectilinear mesh. RT_{60} measurements are consistent with real world expectations, to which the triangular mesh gives the closest match.

The triangular waveguide mesh has been enhanced using both warped and high-resolution techniques, showing an improvement in both cases, being particularly noticeable with the high-resolution mesh when convolved with anechoic audio. Future work will potentially develop and combine these techniques, including the use of a warped high-resolution mesh structure.

9. Acknowledgement

The authors are grateful to Mr. Gui Campos of the University of York for many helpful discussions, Dr. Lauri Savioja of Helsinki University of Technology for advice over Email and Angelo Farina, Associated Professor of Technical Physics at the

University of Parma for the use of the Acoustical Parameters Module of his Aurora software.

10. Refereneces

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