Elmer Circuits

Dr. Eelis Takala, Prof. F. Trillaud

Acknowledgements:
J. Ruokolainen, P. Råback (CSC)
J. Kujanpää (Trafotek Oy)

Contents

- Introduction to Elmer Circuits
- Basic example on how to use the circuit module
- Homogenization technique for stranded coils
- Applications: 3-phase power transformers
- Applications (motivation) : superconducting magnet – Quench model
- Applications: superconducting magnet – Coupling to external circuits
- Parallel simulations of inductive components

Introduction to Elmer Circuits

History

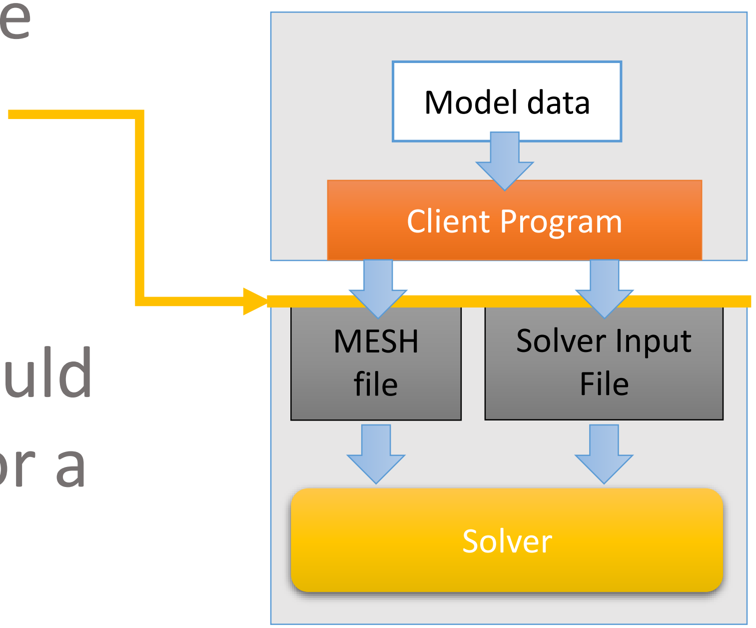
- 2012: Circuit module prototype created
  - Proof of concept with stranded coil
- 2014-2015: Parallax project (TEKES)
  - Objective was to accelerate computation models through parallelization
  - New approach for circuit models: implementation of component sections and possibility to introduce different coil formulations [1]
- 2015-2017: Semtec project (TEKES)
  - Janne Keränen (VTT) as project coordination (Thank you Janne!)
    - One of the greatest projects in Elmerfem history
  - Objective was to develop computational tools for electrical machines using Elmer
  - Circuit module connected for all simulation types (transient/harmonic, 2D/3D)
  - Homogenization models
  - Circuit module published

[1] E. Takala *et. al.* "Parallel Simulations of Inductive Components with Elmer Finite-Element Software in Cluster Environments", Electromagnetics 36(3), April 2016, p. 167-185

Circuit Equation core feature – the interface

- In this presentation, the interface specification of the core feature is presented

- The problem of circuit equation user interface should be addressed by the user (or a client program)
  - There can be multiple implementations



Solvers for the circuit equations: The system matrix

- There are two different ”Solvers”:
  - CircuitsAndDynamics(…)
  - WhitneyAVSolver(…)
- The first one adds the **circuit variable contributions**, **coupling terms** and the **basic circuit equations** to the system matrix (of the WhitneyAVSolver)
- The second one assembles the conventional **AV system matrix** and solves the whole thing

$(\nabla \times a, \nabla \times a')_{\Omega} + (\sigma \partial_t a, a')_{\Omega_c} + (\sigma \nabla v, a')_{\Omega_c}$ AV $(\sigma \partial_t a, \nabla v')_{\Omega_c} + (\sigma \nabla v, \nabla v')_{\Omega_c}$	$+(w I_j, a')_{\Omega_c}$ Circuit variable contributions to the Ampere’s law
$(\partial_t a, w)_{\Omega_{s,j}}$ Coupling terms for the Circuit equations	$I_j R$ Basic Circuit Eqns

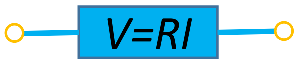
How are the ”basic circuit equations” described in Elmer?

- General equation

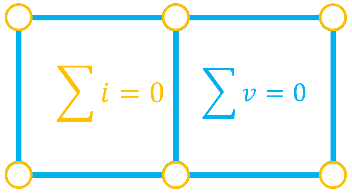
$$\boldsymbol{A}x' + \boldsymbol{B}x = f$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are coefficient matrices, x is the circuit variable vector and f is the source vector

- Circuit equations can be divided into two categories
  1. Component equations (voltage and current relations of a component)
     - Resistor, Inductor, capacitor, etc…



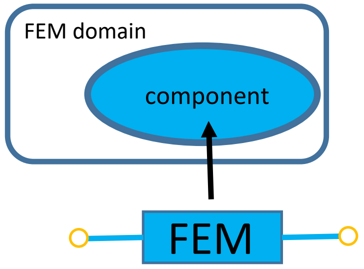
  2. Circuit network equations
     - Kirchoff 1 and 2



Component equations

- An important distinction has to be made in component equations; they can be divided in two groups
  - Linear circuit elements
    - These are the conventional circuit elements arising from the basic circuit theory

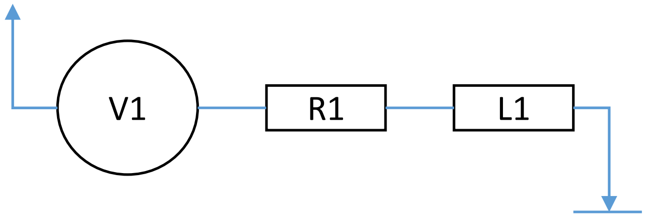
  - FEM components
    - The coupling of circuits and FE model happens through these
    - Created automatically by the FEM solver



Basic example on how to use the circuit module

How are the "basic circuit equations" described in Elmer?

- Example:



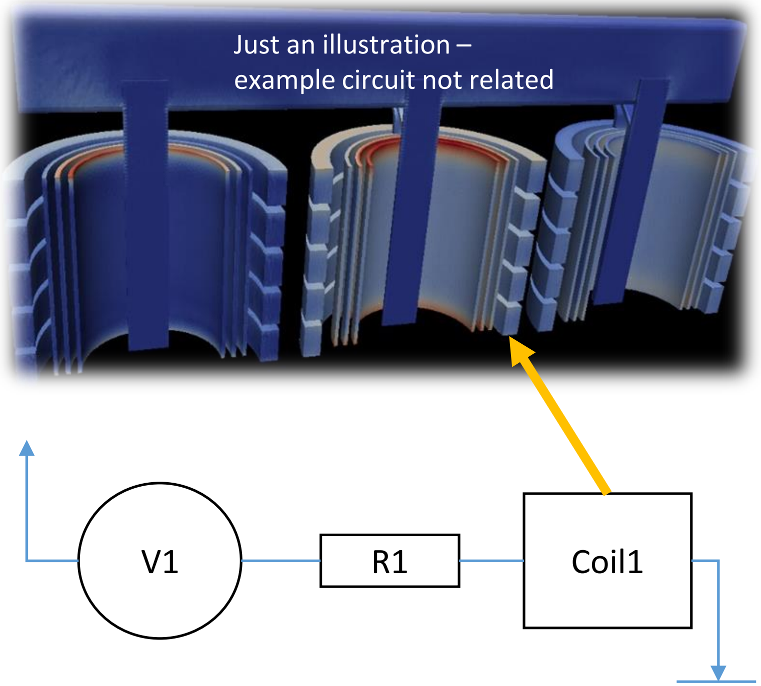
- Network equations:
  - Kirchoff 1:
    - $i_{V1} - i_{R1} = 0$
    - $i_{R1} - i_{L1} = 0$
  - Kirchoff 2:
    - $v_{V1} + v_{R1} + v_{L1} = 0$
- Component equations:
  - V1: $v_{V1} = v_0$
  - R1: $v_{R1} = R_{R1} i_{R1}$
  - L1: $v_{L1} = L_{L1} {i_{L1}}'$

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & L_{L1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} i_{V1} \\ v_{V1} \\ i_{R1} \\ v_{R1} \\ i_{L1} \\ v_{L1} \end{bmatrix}' + \begin{bmatrix} 1 & \mathbf{0} & -1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} & -1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & R_{R1} & -1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & \mathbf{0} & 1 & \mathbf{0} & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -1 \end{bmatrix} \begin{bmatrix} i_{V1} \\ v_{V1} \\ i_{R1} \\ v_{R1} \\ i_{L1} \\ v_{L1} \end{bmatrix} = \begin{bmatrix} 0 \\ v_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{A} \qquad x' \quad + \qquad \boldsymbol{B} \qquad x \quad = \quad f$$

Component Equation Connection to an FE model?

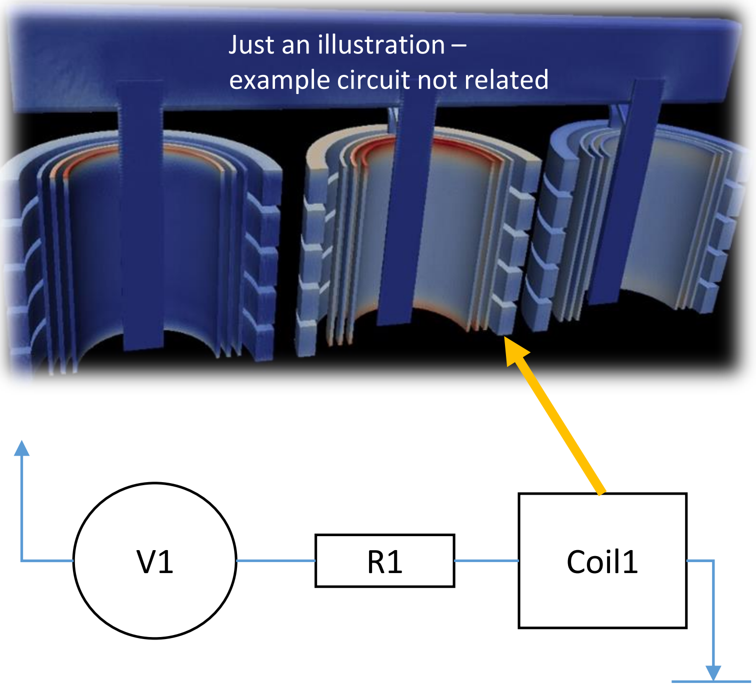
- Elmer can create component equations for bodies of elements in the FE model
- Example: L1 is replaced by a coil component in an FE model
  - Component section needs to be created for ”Coil 1” that is associated to the body that corresponds to the geometry of the coil



$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{\mathrm{V1}} \\ v_{\mathrm{V1}} \\ i_{\mathrm{R1}} \\ v_{\mathrm{R1}} \\ i_{\mathrm{Coil1}} \\ v_{\mathrm{Coil1}} \end{bmatrix}' + \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & R_{\mathrm{R1}} & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{\mathrm{V1}} \\ v_{\mathrm{V1}} \\ i_{\mathrm{R1}} \\ v_{\mathrm{R1}} \\ i_{\mathrm{Coil1}} \\ v_{\mathrm{Coil1}} \end{bmatrix} = \begin{bmatrix} 0 \\ v_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

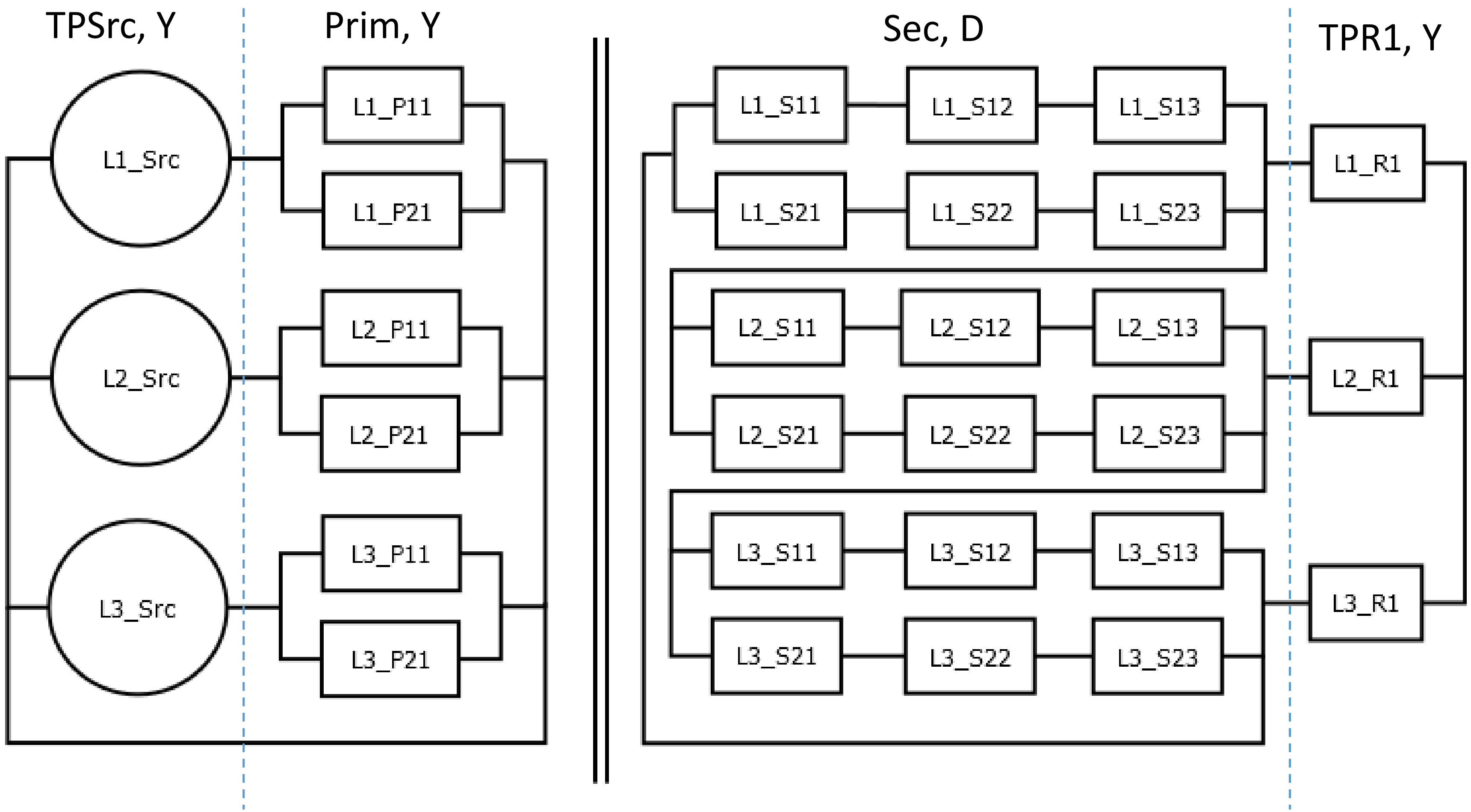
Component Equation
Connection to an FE model?

- The component equation is written by Elmer!!! (In this example the red number 1 in matrix ***B***)
    - FEM component equation is always written to the voltage row
    - Circuit variable contributions and coupling terms to the AV system matrix are written as well (these describe the component actually)
    - Note that in principle Elmer could write also the circuit element equations!



$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{\mathrm{V1}} \\ v_{\mathrm{V1}} \\ i_{\mathrm{R1}} \\ v_{\mathrm{R1}} \\ i_{\mathrm{Coil1}} \\ v_{\mathrm{Coil1}} \end{bmatrix}' + \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & R_{\mathrm{R1}} & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{\mathrm{V1}} \\ v_{\mathrm{V1}} \\ i_{\mathrm{R1}} \\ v_{\mathrm{R1}} \\ i_{\mathrm{Coil1}} \\ v_{\mathrm{Coil1}} \end{bmatrix} = \begin{bmatrix} 0 \\ v_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Parametric circuit model (A relatively simple example, Yd transformer)

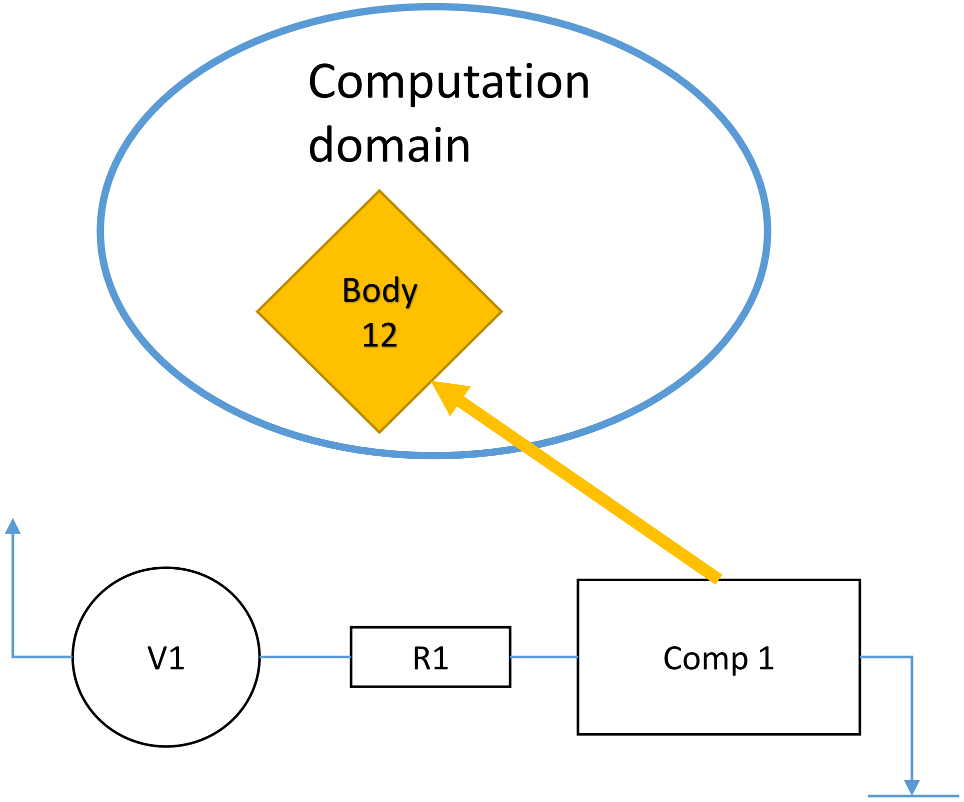


How does it work in practice? component – body association

Solver input file (SIF):

```
…
Body 12
  Name=Coil
  Target Bodies(1) = $ Coil
  Equation = 2
  Material = 5
  Initial Condition = 1
  Body Force = 1
End

Component 1
  Name=Stranded Coil
  Master Bodies(1) = 12
  Coil Type = String Stranded
  Number of Turns = Integer 14
  Electrode Area = Real $ 2376e-6
End
```



How does it work in practice? component – body association

circuits.definitions:

$ Circuits = 1

$ C.1.variables = 6
$ C.1.name.1 = "i_v1"
$ C.1.name.2 = "v_v1"
$ C.1.name.3 = "i_R1"
$ C.1.name.4 = "v_R1"
$ C.1.name.5 = "i_component(1)"
$ C.1.name.6 = "v_component(1)"

! i_v1 – i_R1 = 0
$ C.1.B(0,0) = 1
$ C.1.B(0,2) = -1

! v_v1 = v0
$ C.1.B(1,1) = 1
$ C.1.source.2 = "v0"
…
! R1 * i_R1 –v_R1 = 0
$ C.1.B(3,2) = R1
$ C.1.B(3,3) = -1
!etc…

Both circuit variables need to be declared. The component behaviour is described by the equation that is automatically written (by the solver) for the v_component

The other equation (that describes the components relation to the circuit) needs to be written in the matrices A and B

SIF:

…
Body Force 1
  Name = "Circuit"
…
! Phase 1
v0 = Variable time
   Real Procedure "phasecurrents" "v0"
…
End
…

Note that this is actually a component equation! So we could let elmer write it if we wanted

Non-linear basic component (specification)

circuits.definitions:

```
$ Circuits = 1

$ C.1.variables = 6
$ C.1.name.1 = "i_v1"
$ C.1.name.2 = "v_v1“
$ C.1.name.3 = “i_component(2)”
$ C.1.name.4 = “v_component(2)”
$ C.1.name.5 = "i_component(1)"
$ C.1.name.6 = "v_component(1)“

! i_v1 – i_R1 = 0
$ C.1.B(0,0) = 1
$ C.1.B(0,2) = -1

! v_v1 = v0
$ C.1.B(1,5) = 1
$ C.1.source.2 = “v0“

!etc…
```

Solver input file (SIF):

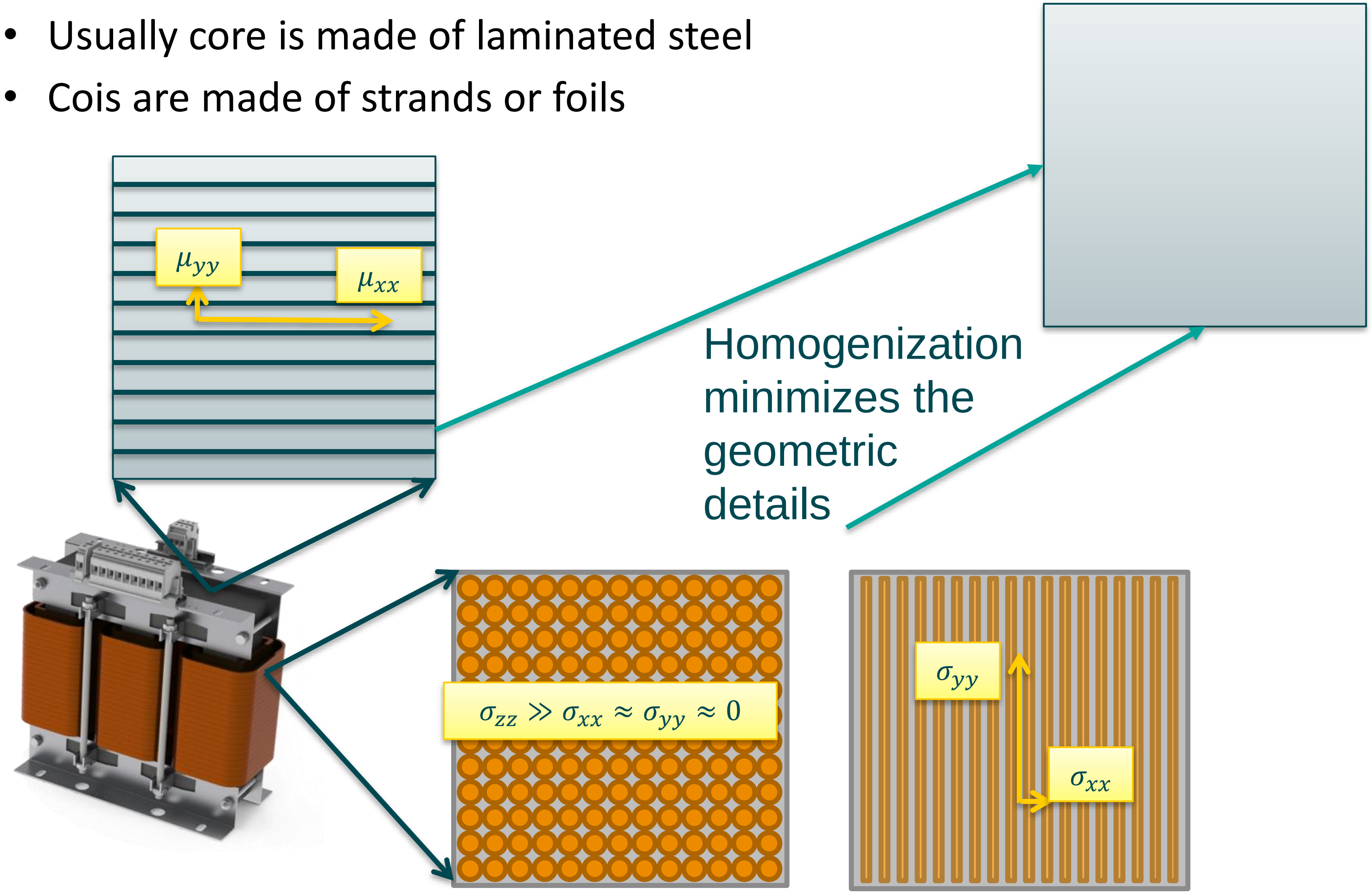
```
Component 2
  Name=R1
  Type = Resistor
  Resistance = Variable T1
    Real Procedure “Components” “getResistance”
End


…
Body Force 1
  Name = "Circuit"
…
! Phase 1
v0 = Variable time
    Real Procedure "phasecurrents" “v0"
…
End
…
```

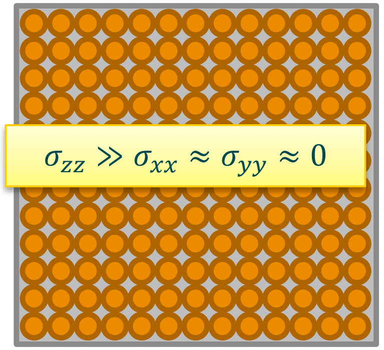
NOTE: This type of resistor has not been implemented yet but will be done in the future

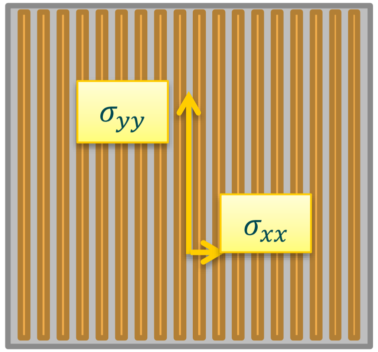
Homogenization techniques

- Usually core is made of laminated steel
- Cois are made of strands or foils



Elmer coil models






```
Component 1
  Name=Massive Coil
  Master Bodies(1) = 1
  Coil Type = String Massive
End
```

- A natural coupling with the AV formulation
- Strength: "perfect behaviour (losses correctly computed)"
- Drawback: difficult to apply to anything (applications are detailed)

```
Component 1
  Name=Stranded Coil
  Master Bodies(1) = 1
  Coil Type = String Stranded
  Electrode Area = Real $area
End
```

- Strength: practical, easy to use
- Drawback: needs to be equipped with a homogenization model in order to compute losses

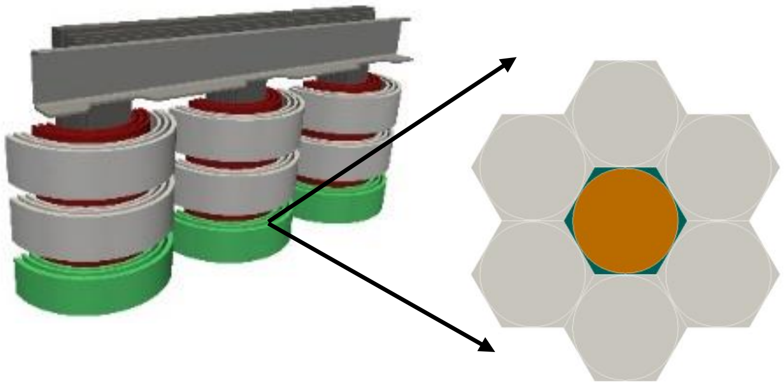
```
Component 1
  Name=Stranded Coil
  Master Bodies(1) = 1
  Coil Type = String Foil Winding
  Winding thickness
  Foil Winding Voltage
Polynomial Order = Integer 2
End
```

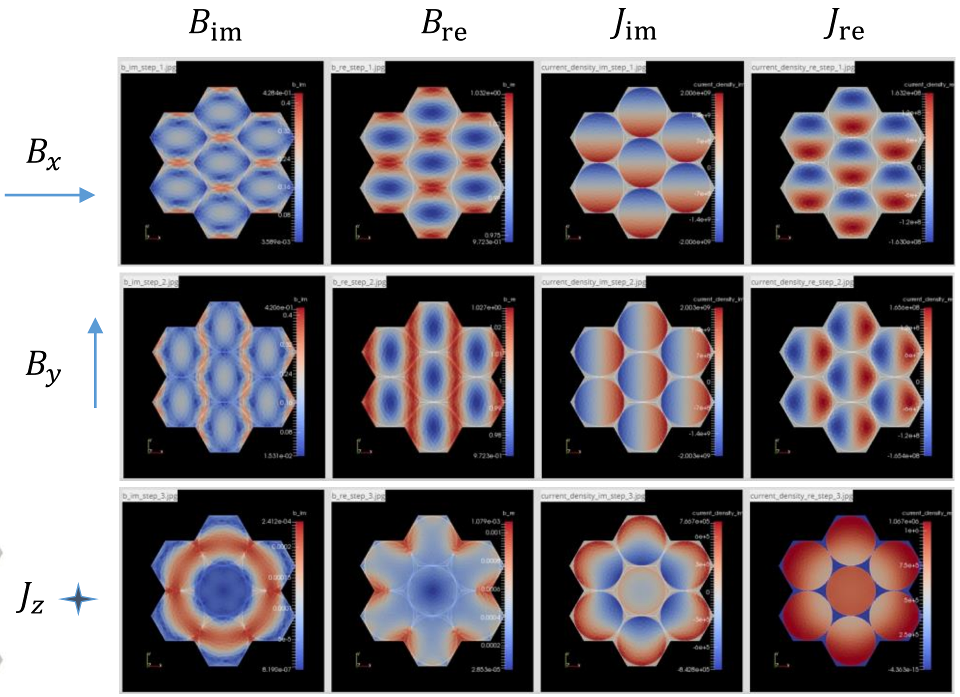
- Strength: practical, easy to use, computes losses correctly in low frequencies
- Drawback: current distribution is assumed homogenic over the foil thickness -> needs a homogenization model in high frequencies

Homogenization technique for stranded coils

homogenization model (stranded coil) – similar with [6][7]

- Assuming the coil consists of a periodic coil structure
  - Make a small model of the periodic structure
  - Compute the response of one cell
  - Use the response in the homogenization model





[6] J. Gyselinck and P. Dular, ”Frequency-Domain Homogenization of Bundles of Wires in 2-D Magnetodynamic FE Calculations”, IEEE Trans. Magn., 41(5), May 2005

[7] G. Meunier, A. T. Phung, O. Chadebec, X. Margueron, J. Keradec. ”Proprietes macroscopiques equivalentes pour representer les pertes dans les bobines conductrices”, Revue Internationale de G´enie Electrique, 2008, 11 (6), pp.675-694

Example

- First the ”material response” is computed

Activate homogenization parameter computation

```
Simulation 1
  ....
  Coordinate System = Cartesian
  Simulation Type = Scanning
  Timestep Intervals = 3
  Timestep sizes = 1
  Additive Namespaces = Logical True
  ....
End
```

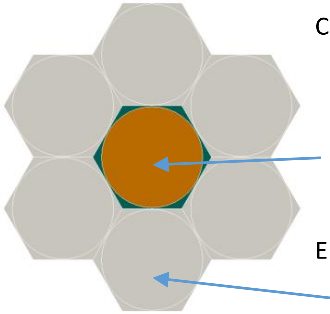
```
Solver 4  !---- MagnetoDynamics2D, bSolver
  ....
  Procedure = "MagnetoDynamics2D" "bSolver"
  Calculate Homogenization Parameters = Logical True
  ....
End
```

Solution settings

```
Boundary Condition 1  !---- none core
  Target Boundaries(18) = 1 2 6 7 8 9 \
              12 13 14 17 20 21 \
              24 25 26 28 29 30
  Coupled 1: Magnetic Flux Density 1 = 1
  Coupled 2: Magnetic Flux Density 2 = 1
  Coupled 3: A re = Real 0
  Coupled 3: A im = Real 0
End
```

```
Body Force 1  !---- Circuit
  Name = "Circuit"
  S Re = Real 0
  Coupled 3: S Re = Real 1
  S Im = Real 0.0
End
```

Output settings



```
Component 5  !---- wp1_5
  Name = String wp1_5
  Master Bodies(2) = Integer 5 12
  Coil Type = String massive
  Coupled 1: Homogenization Reluctivity Output Component = String 11
  Coupled 2: Homogenization Reluctivity Output Component = String 22
  Coupled 3: Homogenization Conductivity Output Component = String 33
End
```

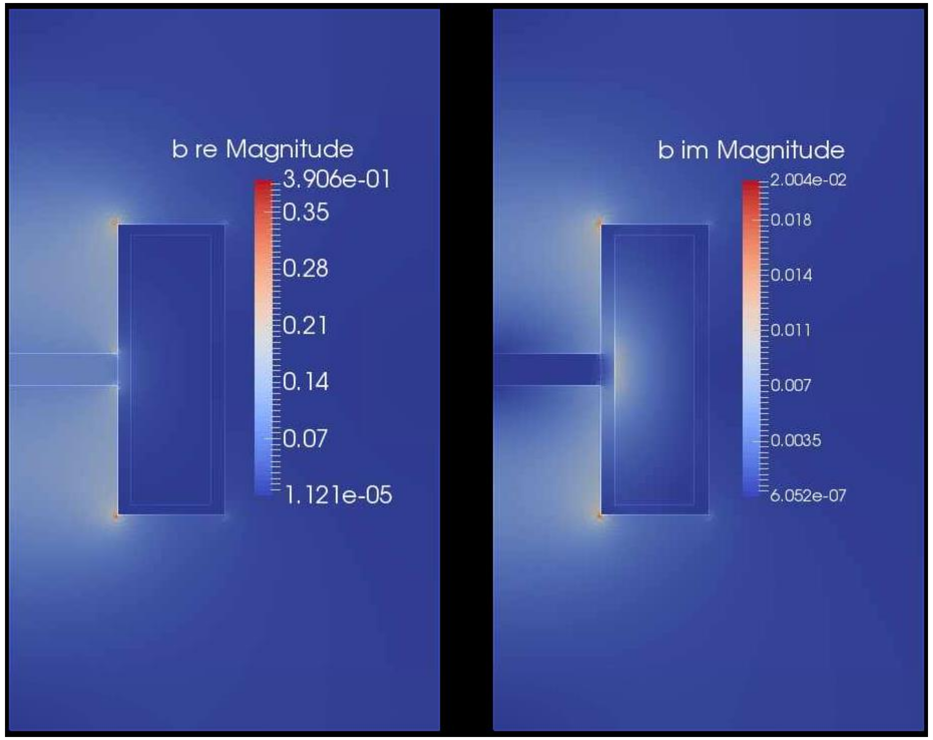
Other components can to be defined without homogenization output

Coupled i are the 3 elementary solutions!

Example

- Now the material response can be used in coil component

```
Component 1  !---- wp1
  ....
  Master Bodies = Integer 3
  Coil Type = String stranded
  Homogenization Model = Logical True
  Nu 11 = Real 821526.5625
  Nu 11 im = Real 289980.15625
  Nu 22 = Real 821526.5625
  Nu 22 im = Real 289980.15625
  Sigma 33 = Real 47728506.5296
  Sigma 33 im = Real -12454099.5408
  ....
End
```



Example results

- DC Power of the test coil 0.2061W
- AC Power 10kHz
  - Massive 11.57W (Fig. 1)
  - Homogenization 11.88W (Fig. 2)
- Capasitive effects are not taken into account in either of the models
- Only applicable to structures with linear permeability

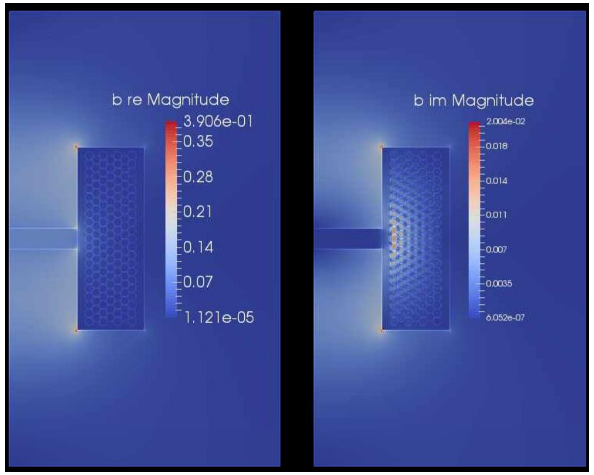


Fig 1. Massive

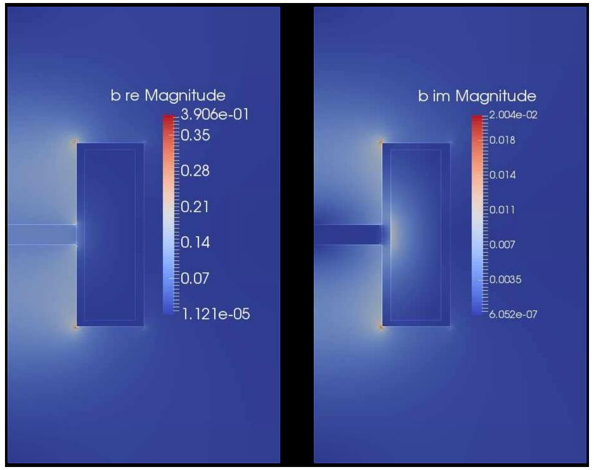
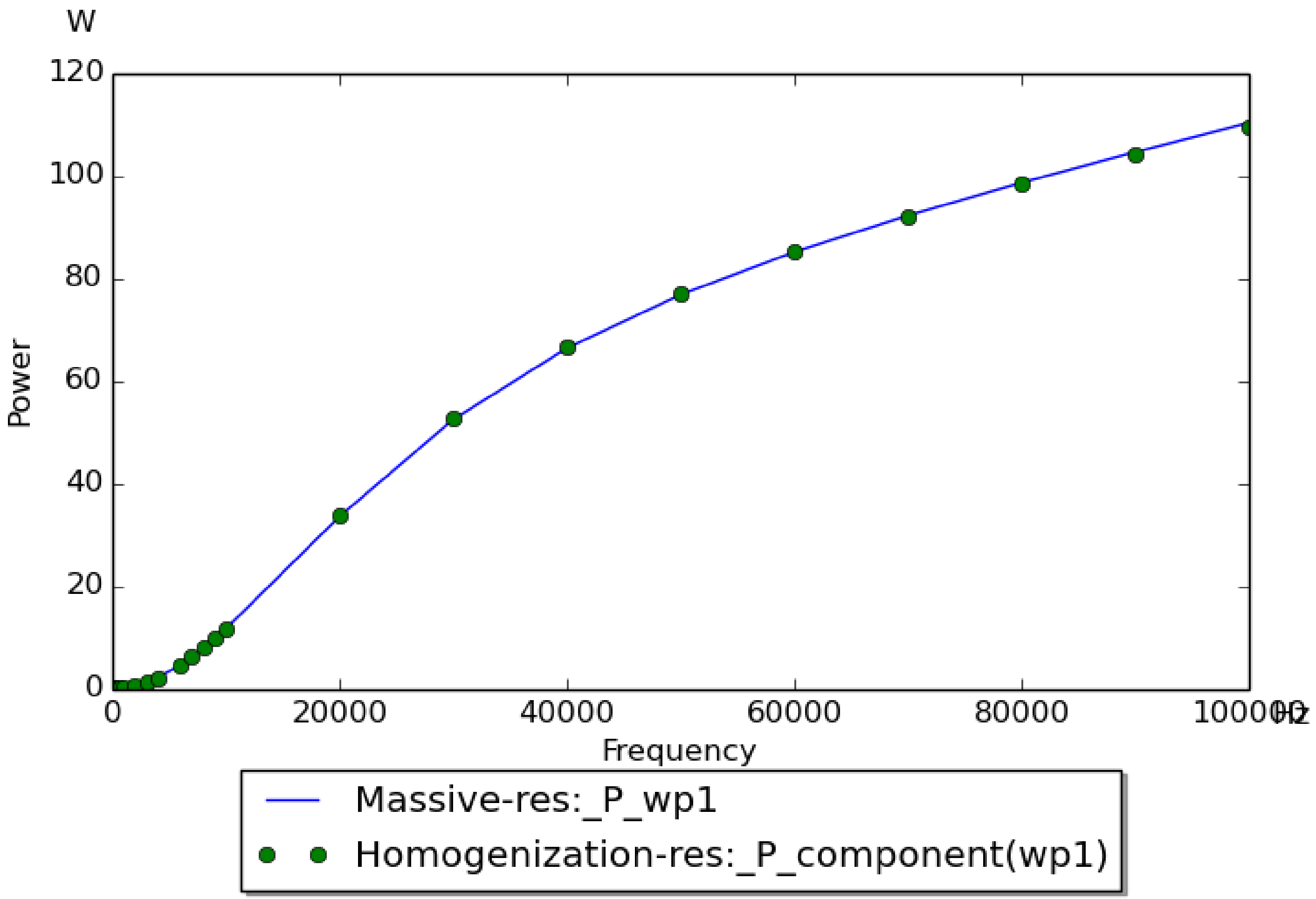


Fig 2. Homogenization

Massive vs. Homogenization 1-100kHz



Applications: 3-phase power transformers

Power transformers - introduction

- Transformer short circuit behaviour has been an academic interest at least for a half a century [1]
- The first foil winding model by N. Mullineux [2]
  - Analytical but involved numerical integration
  - ”less than a minute” with Atlas super computer (cost: 500 pounds/hour)

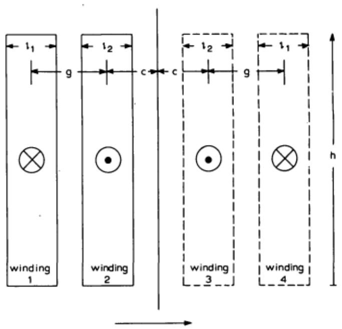


Fig. 1
Image system for the mathematical model

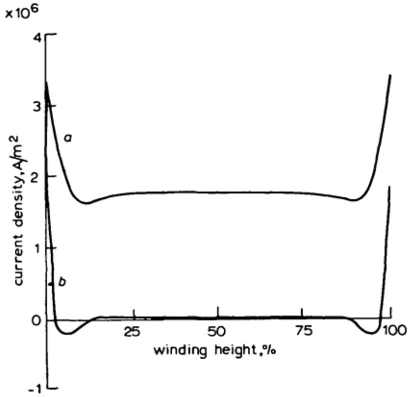


Fig. 3
Current-density distribution
a Component in phase with h.v. current
b Component in quadrature (leading positive)

Image source: https://en.wikipedia.org/wiki/Atlas_(computer)

[1] P. L. Dowell, ”Effects of eddy currents in transformer windings”, Proc. IEE., vol. 113, no. 3, pp. 1387-1394, 1966
[2] N. Mullineux, J. R. Reed and I. J. Whyte, ”Current distribution in sheet and foil-wound transformers”, Proc. IEE., vol. 116, no. 1, pp. 127-129, January 1969

Power transformers - introduction

- Nowadays a fast and decent 2D harmonic FEM short circuit model can be run in a matter of seconds

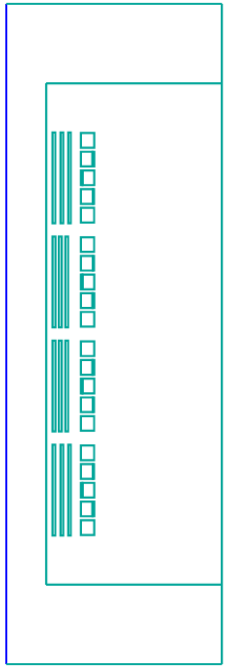
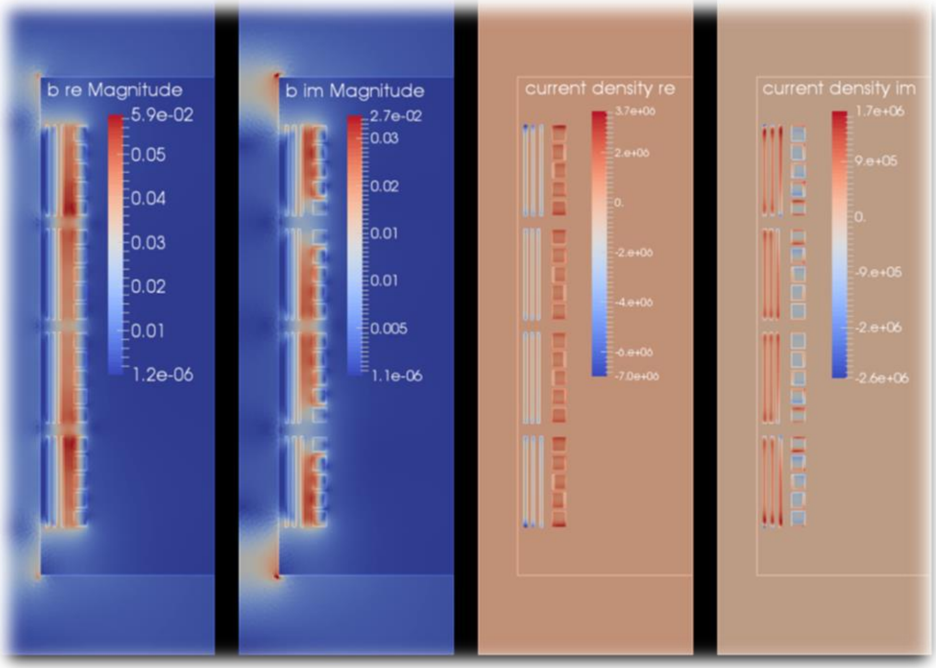
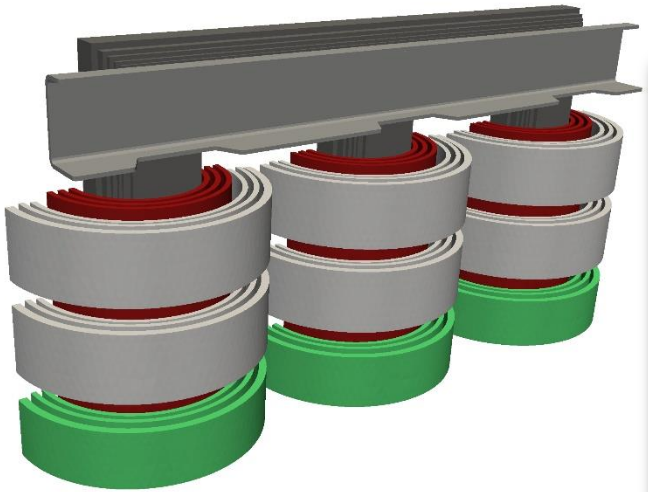


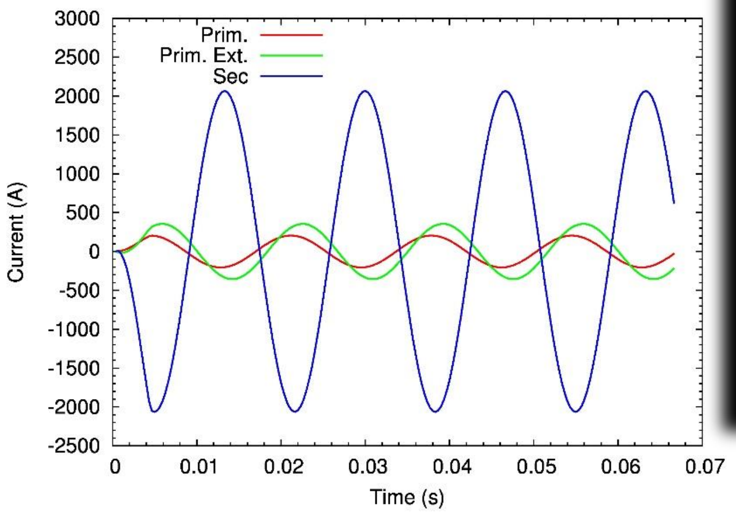
Image source: https://en.wikipedia.org/wiki/ThinkPad

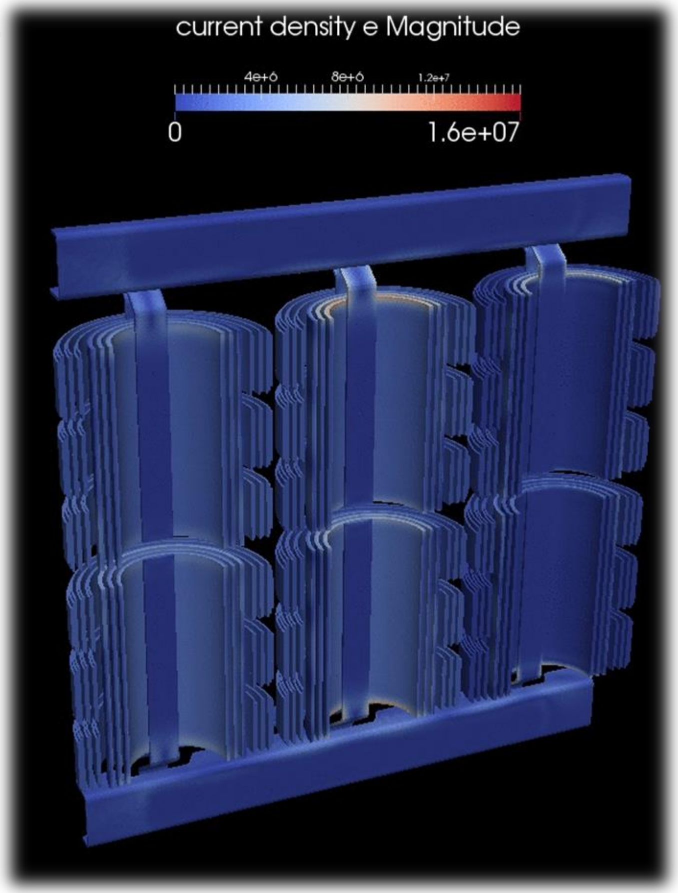


3D Example

- Apply the A-V formulation on a test transformer
- Geometry
  - Primary (Stranded)
  - Primary phase shift (30 deg)
  - Secondary (Foil Winding)
  - Core (mu: 2e3)
  - Clamp (sigma: 1e7)
  - Air box (Covers everything)

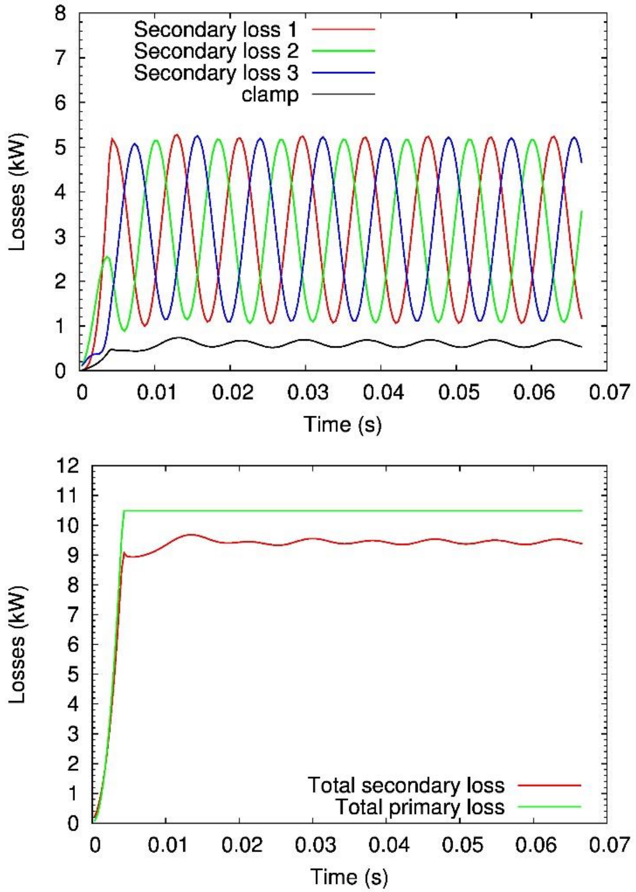


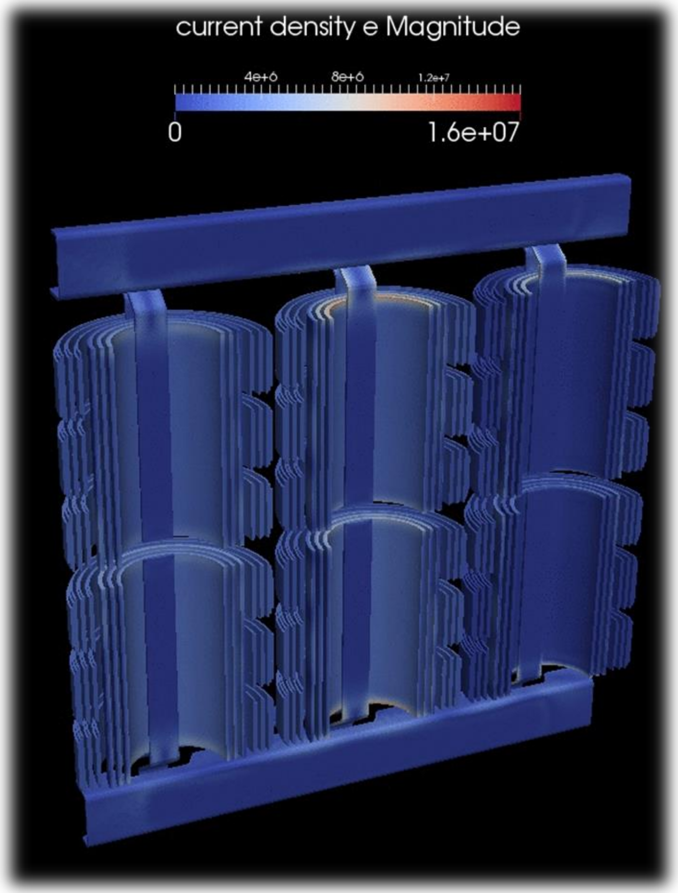




Loss estimation

- Now we can compute the losses due to the uneven distribution of current at any point at a given time
- Or with a little touch of paraview we can produce really cool animations…





Applications (motivation) : superconducting magnet – Quench model

Quench model by courtesy of Prof. F. Trillaud from UNAM

The simulation shows the motivation for quench protection and thus the connection to external circuits

Development for superconducting magnets

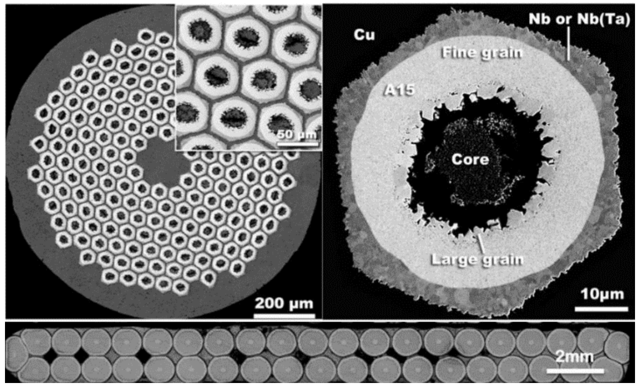
- Superconductors = high current density at no loss at cryogenic temperatures
- Superconducting magnets are prone to “quenches”
- A quench is the sudden appearance of local Joule dissipation followed by a propagating heat wave through the magnet => multiphysics model coupling magnetodynamic solver and heat solver
- A quench is harmful => need of rapid protection and detection systems => circuit model

Steps of quench process:
1: Initial normal zone (first localized dissipation
2-3: Diffusion (expansion of the dissipative sone)
4: Propagation (dissipative front at constant velocity)

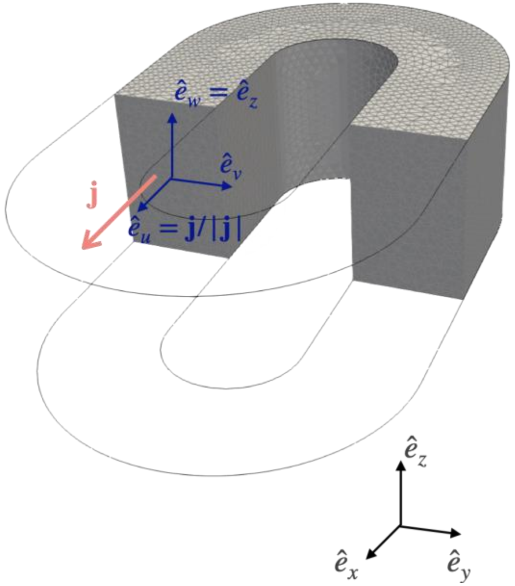
F. Trillaud, “Etude de la stabilité thermoélectronique des conducteurs supraconducteurs à basse température critique et contribution à l'étude de la stabilité thermoélectrique des supraconducteurs à haute température critique.”, PhD thesis, 2005, Laboratoire de Génie Electrique de Grenoble

Coil model

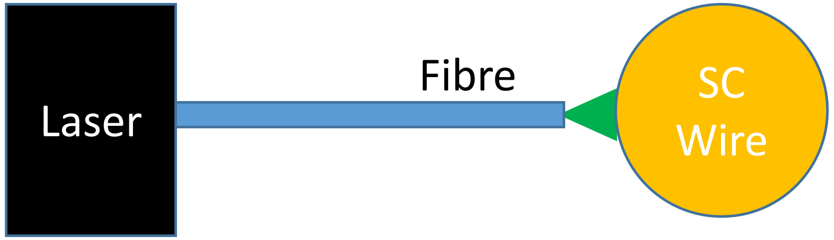
- Coil wound with composite material => anisotropic material properties
- Heat conductivity tensor in a local coordinate system => UDF
- CoilSolver to model a closed coil



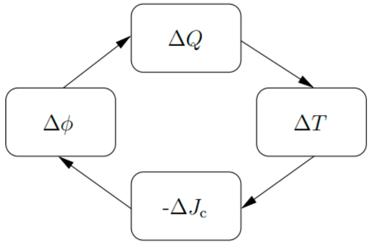
Reviews of Accelerator Science and Technology, Vol. 05, pp. 25-50 (2012)



Initiating a quench in a superconducting strand can be done with a very low energy



- Minumum perturbation energy to quench a Nb3Sn SC wire in between 0.5uJ - 23uJ
  - Kinetic energy of a water drop (50 uL or 50mg) falling 1 mm – 47 mm



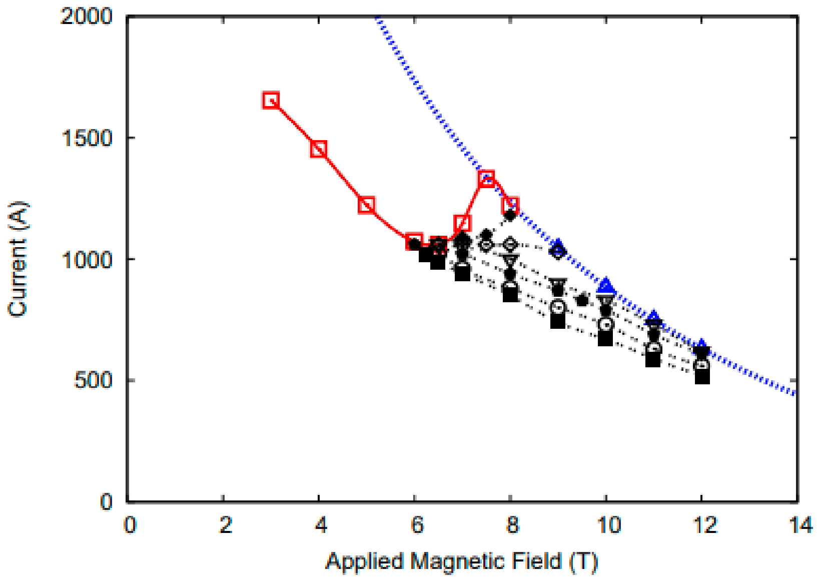
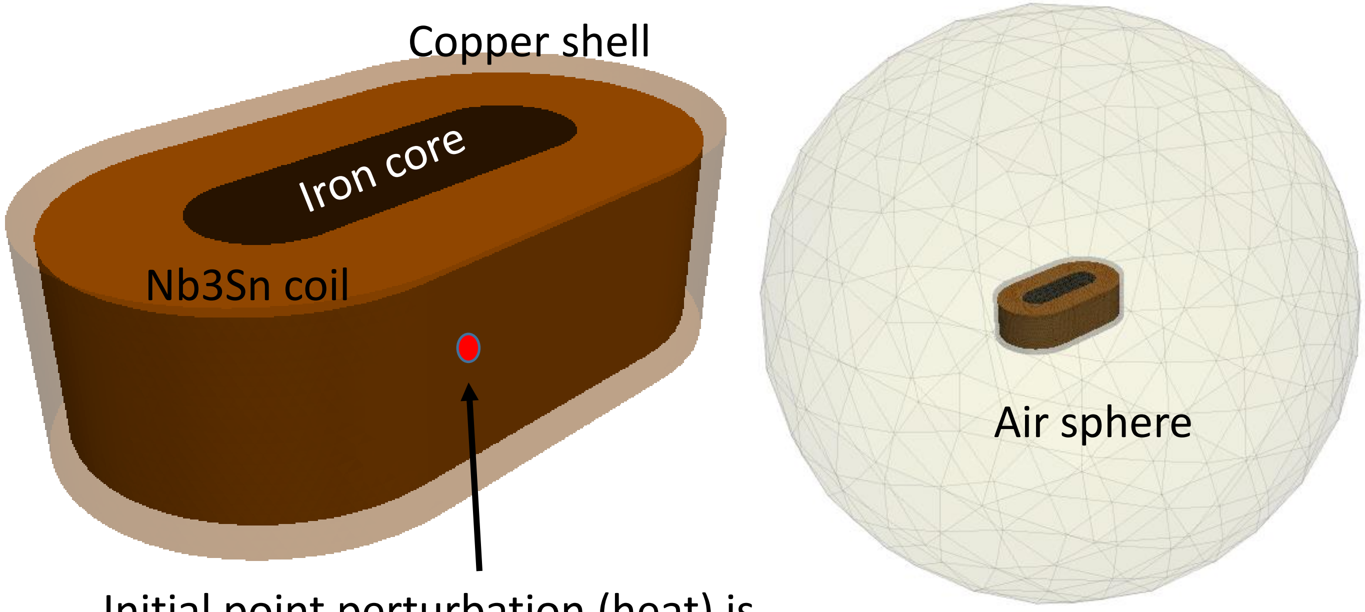


Figure 24. Full characterization of sample 1 with RRR of 129 at a) 1.9 K and b) 4.3 K. (▴) represents the critical current measurements, (····) is the critical surface, (▫) and (▴) represent the quench current by natural perturbation in V-I and V-H, respectively. Laser quenching data is represented by (•) 0.5 μJ, (⋄) 1 μJ, (▿) 4.4 μJ, (•) 7 μJ, (▴) 10 μJ, (◦) 13 μJ, (▪) 23 μJ. **[P4]**

E. Takala, "THE LASER QUENCHING TECHNIQUE FOR STUDYING THE MAGNETO-THERMAL INSTABILITY IN HIGH CRITICAL CURRENT DENSITY SUPERCONDUCTING STRANDS FOR ACCELERATOR MAGNETS", PhD thesis, 2012, University of Turku

Quench model

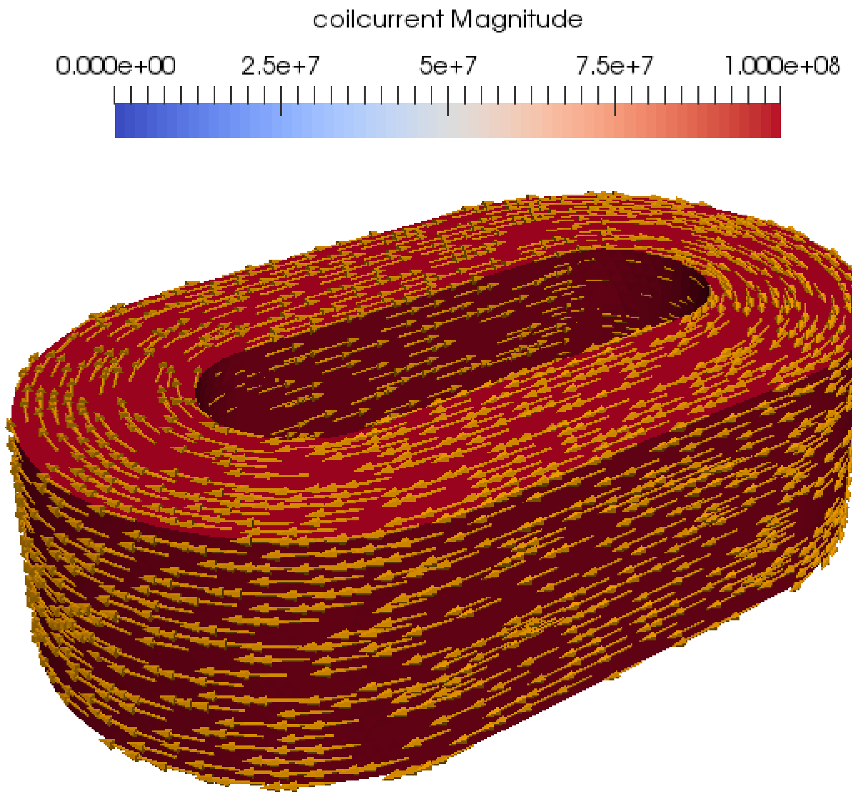
- Quench modelled through a heat source
- The heat source depends on time, temperature, magnetic flux density, and current:

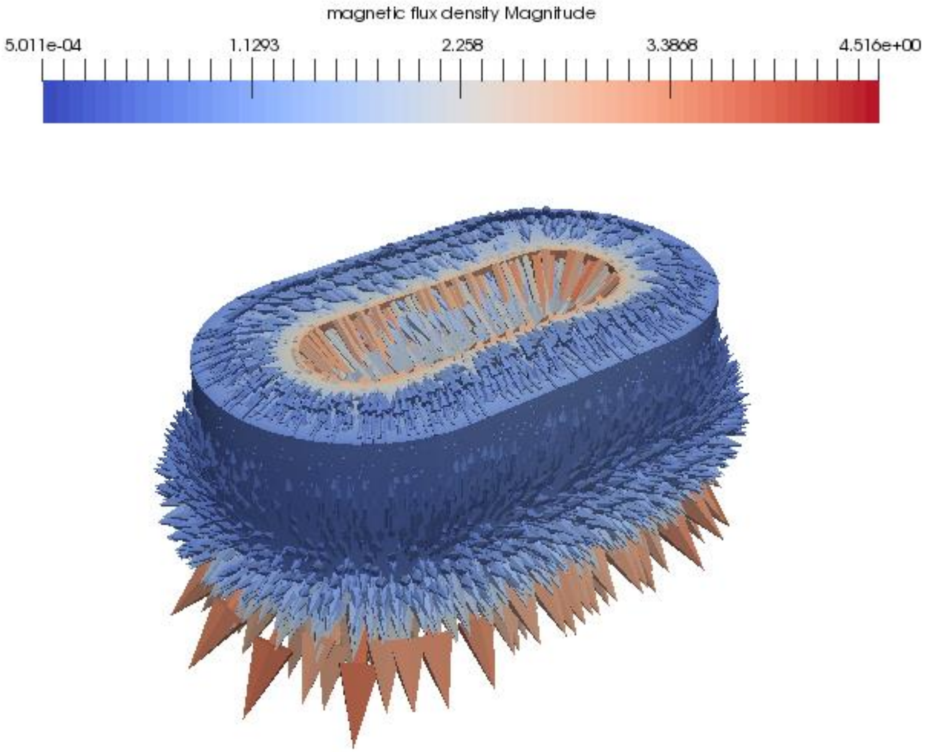


Initial point perturbation (heat) is given at the center of the straight section

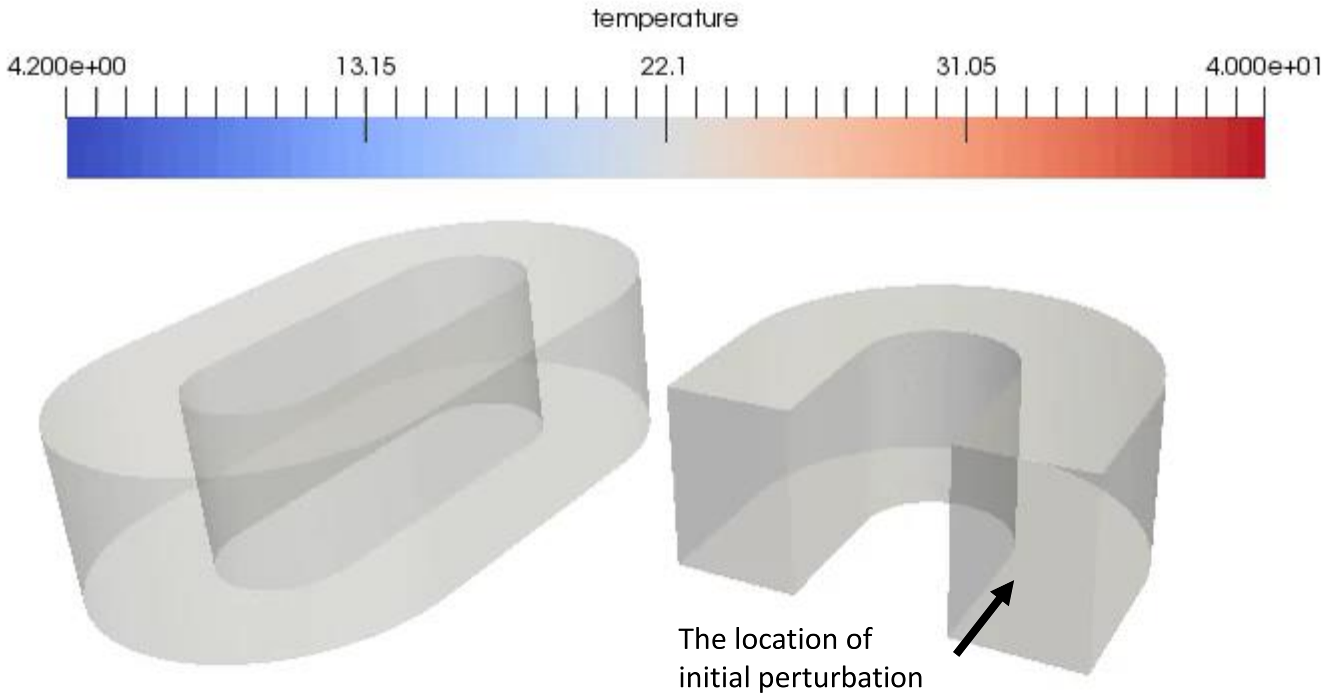
Heat Source = Variable Time, Temperature, Magnetic Flux Density, coilcurrent
Real Procedure "./Fortran90/dissipation" "getDissipation"

Current (and thus magnetic flux density) is assumed constant





Example (problem of ”quench”)

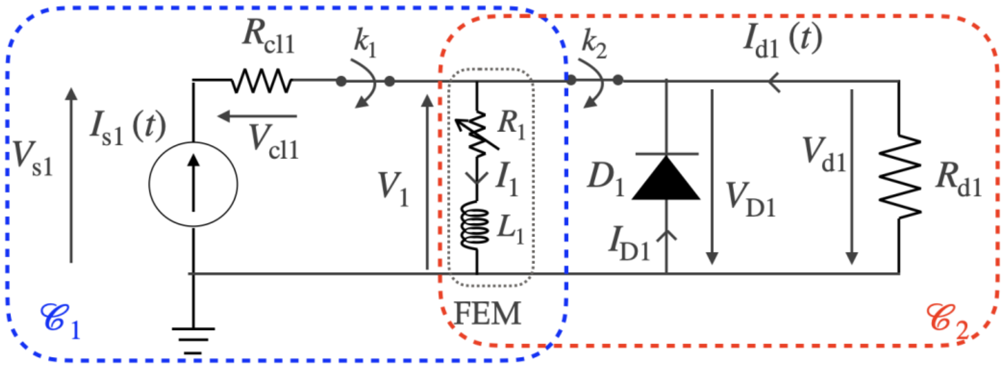


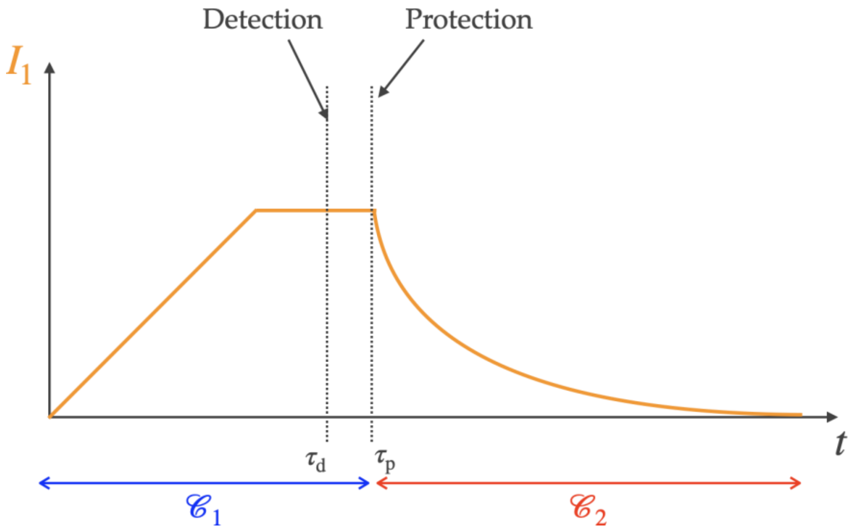
- Note that the perturbation only lasts for 10 simulation steps (out of 100) => heating mainly due to the propagating normalzone
- Current is assumed constant here => if nothing is done, the temperature keeps rising until something breaks => protection system => circuit coupling

Applications: superconducting magnet – Coupling to external circuits

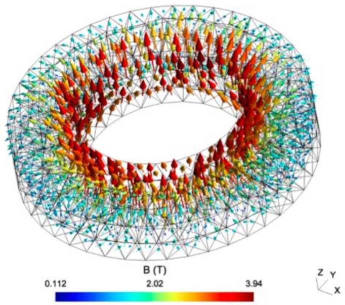
External electrical circuit

- Model of the protection system through an external circuit
- Presence of nonlinear lumped-parameters components (diode)
- Coupling for a stranded coil model through current source with the “CoilSolver” => closed coil with enforced “div J = 0”

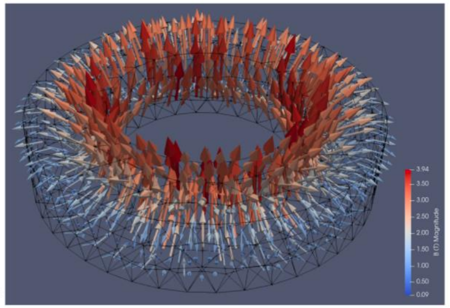




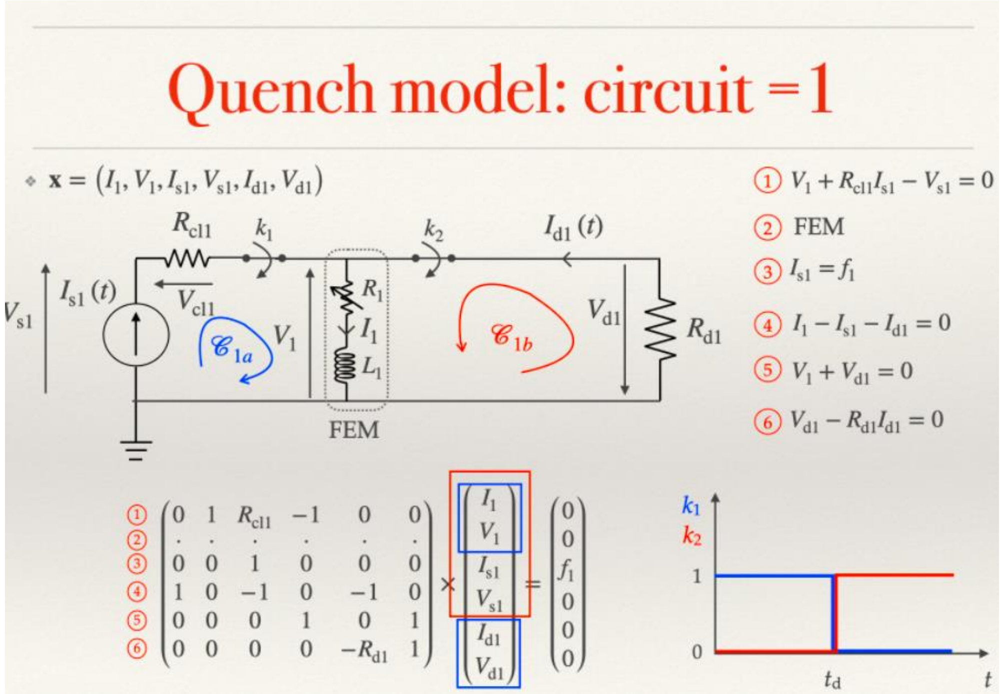
Example: external coupling with a superconducting coil



Onelab (Gmsh/GetDP)
https://onelab.info/



Elmerfem
http://www.elmerfem.org/blog/

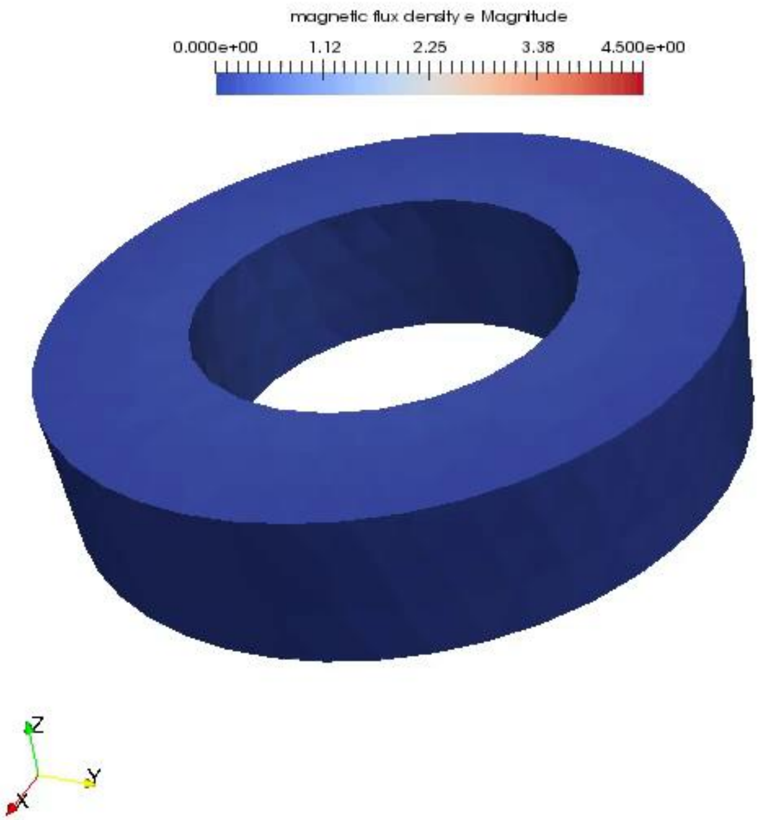


- Three simulation phases:
  1. Ramp-up
  2. Hold
  3. Extraction

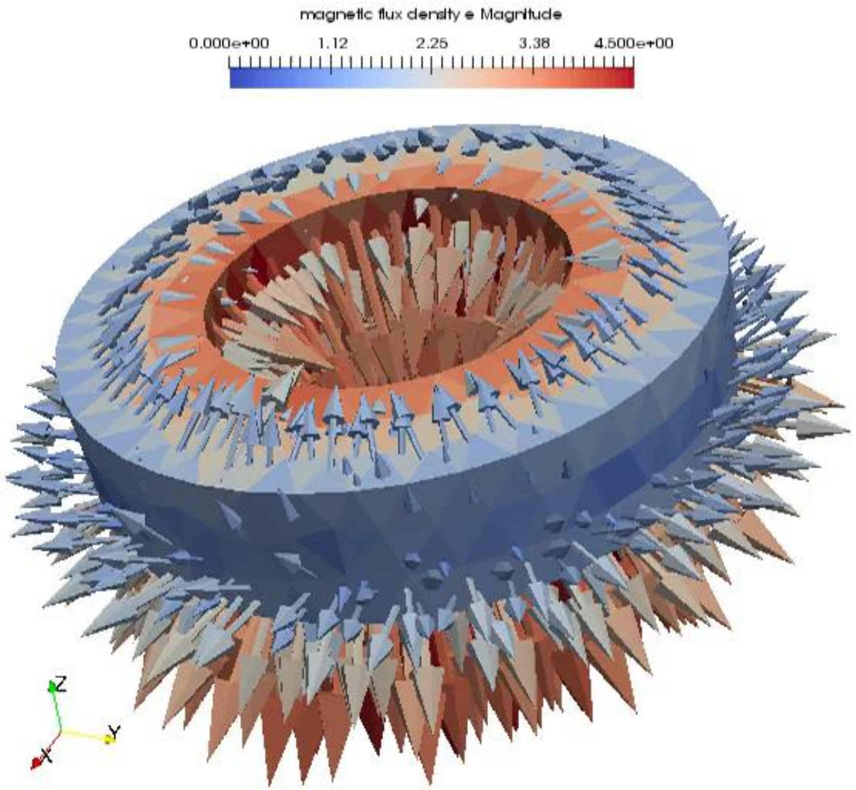
Follow the development at: https://github.com/ettaka/elmer-elmag/tree/main/SuperconductingCoilCircuit

Result animations

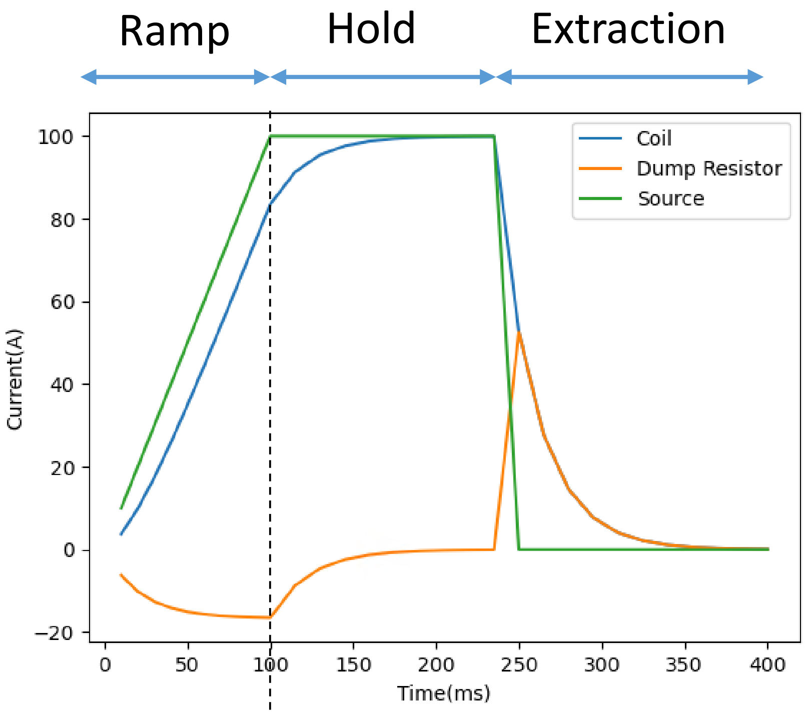
Ramp-up



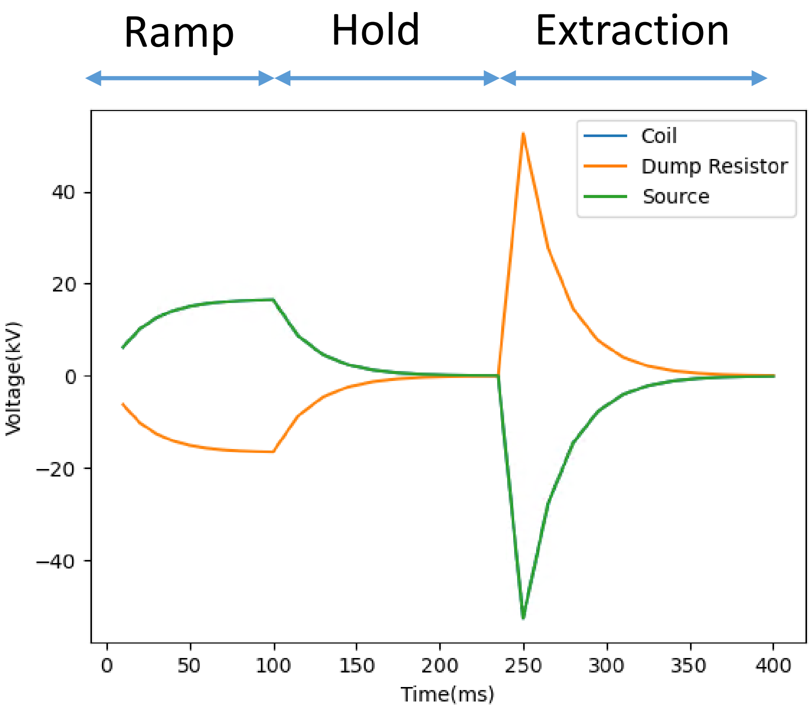
Hold and extraction



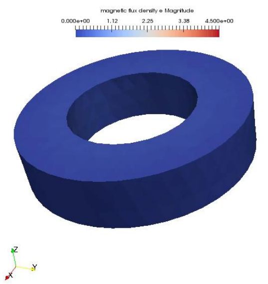
Results

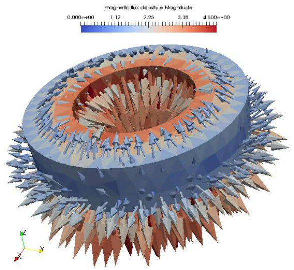


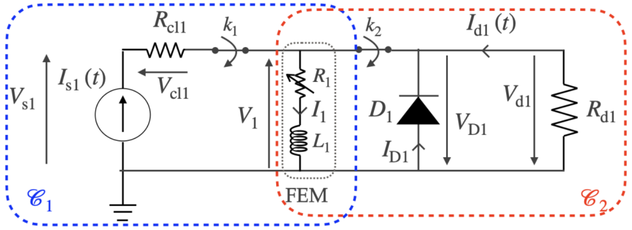
Ramp
Hold
Extraction
Coil
Dump Resistor
Source
Current(A)
Time(ms)
100
80
60
40
20
0
−20
0
50
100
150
200
250
300
350
400



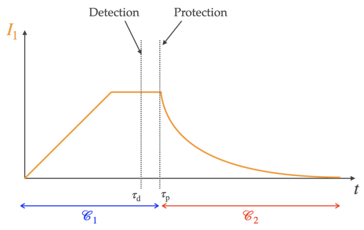
Ramp
Hold
Extraction
Coil
Dump Resistor
Source
Voltage(kV)
Time(ms)
40
20
0
−20
−40
0
50
100
150
200
250
300
350
400







Rcl1
k1
k2
Id1 (t)
Vs1
Is1 (t)
Vcl1
V1
R1
I1
L1
D1
ID1
VD1
Vd1
Rd1
FEM
𝒞1
𝒞2



Detection
Protection
I1
t

Parallel simulations of inductive components

[1] E. Takala *et. al.* ”Parallel Simulations of Inductive Components with Elmer Finite-Element Software in Cluster Environments”, Electromagnetics 36(3), April 2016, p. 167-185

Circuit Equations

- The new circuit equation module supports easy addition of new formulations
- So far three implemented (supports all simulation types)
  - Massive inductors as in [9]

$$(\nabla \times a, \nabla \times a')_\Omega + (\sigma \partial_t a, a')_{\Omega_c} + \sum_{i \in \Gamma_j} V_i \, (\sigma \nabla v_0^i, a')_{\Omega_c} = 0, \forall a' \in F_a(\Omega)$$

$$(\sigma \partial_t a, \nabla s^i)_{\Omega_c} + V_i (\sigma \nabla v_0, \nabla s^i)_{\Omega_c} = I_i, \text{ with } v_0 = s^i = \sum_{n \in \Gamma_j^i} s_n$$

  - Stranded inductors as in [10]

$$(\nabla \times a, \nabla \times a')_\Omega + (\sigma \partial_t a, a')_{\Omega_c} = (w I_j, a')_{\Omega_c}, \forall a' \in F_a(\Omega)$$

$$(\partial_t a, j_{s,j})_{\Omega_{s,j}} + I_j (\sigma^{-1} j_{s,j}, j_{s,j})_{\Omega_{s,j}} = -V_j$$

  - Foil windings as in [11] (voltage $V(\alpha)$ is dependent on the thickness of the coil)

$$(\nabla \times a, \nabla \times a')_\Omega + (\sigma \partial_t a, a')_{\Omega_c} + \sum_{i \in \Gamma_j} (\sigma V'(\alpha) \nabla v_0^i, a')_{\Omega_c} = 0, \forall a' \in F_a(\Omega)$$

$$(\sigma \partial_t a, V'(\alpha) \nabla s^i)_{\Omega_c} + (\sigma V(\alpha) \nabla v_0, V'(\alpha) \nabla s^i)_{\Omega_c} = \frac{N_f}{L_\alpha} I_i \int_{\Omega_\alpha} V'(\alpha), \text{ with } v_0 = s^i = \sum_{n \in \Gamma_j^i} s_n$$

[9] P. Dular, F. Henrotte, W. Legros, "A General Natural Method to Define Circuit Relations Associated with Magnetic Vector Potential Formulations", IEEE Trans. Magn. 35(3) May 1999

[10] P. Dular et. Al. "Dual Complete Procedures to Take Stranded Inductors into Account in Magnetic Vector Potential Formulations", IEEE Trans. Magn. 36(4) July 2000

[11] P. Dular, C. Geuzaine, "Spatially Dependent Global Quantities Associated With 2-D and 3-D Magnetic Vector Potential Formulations for Foil Winding Modeling", IEEE Trans. Magn. 38(2) May 2002

Computational performance

- Global matrix structure changes due to circuit equations!

$$\begin{bmatrix} \boldsymbol{a}_{aa} & \boldsymbol{a}_{av} \\ \boldsymbol{a}_{va} & \boldsymbol{a}_{vv} \end{bmatrix} \begin{bmatrix} \vec{a}_{\mathrm{e}} \\ \vec{v}_{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} \vec{f}_a \\ \vec{f}_v \end{bmatrix} \longrightarrow \begin{bmatrix} \boldsymbol{a}_{aa} & \boldsymbol{a}_{av} & \boldsymbol{a}_{aV} & \\ \boldsymbol{a}_{va} & \boldsymbol{a}_{vv} & & \\ \boldsymbol{a}_{Va} & & \boldsymbol{a}_{VV} & \boldsymbol{a}_{VI} \\ & & \boldsymbol{a}_{IV} & \boldsymbol{a}_{II} \end{bmatrix} \begin{bmatrix} \vec{a}_{\mathrm{e}} \\ \vec{v}_{\mathrm{n}} \\ \vec{V}_{\mathrm{global}} \\ \vec{I}_{\mathrm{global}} \end{bmatrix} = \begin{bmatrix} \vec{f}_a \\ \vec{f}_v \\ \\ \vec{f}_I \end{bmatrix}$$

- Depending on the computed problem, the global variables may couple any elements
  - May cause severe communication problems in parallel computing!

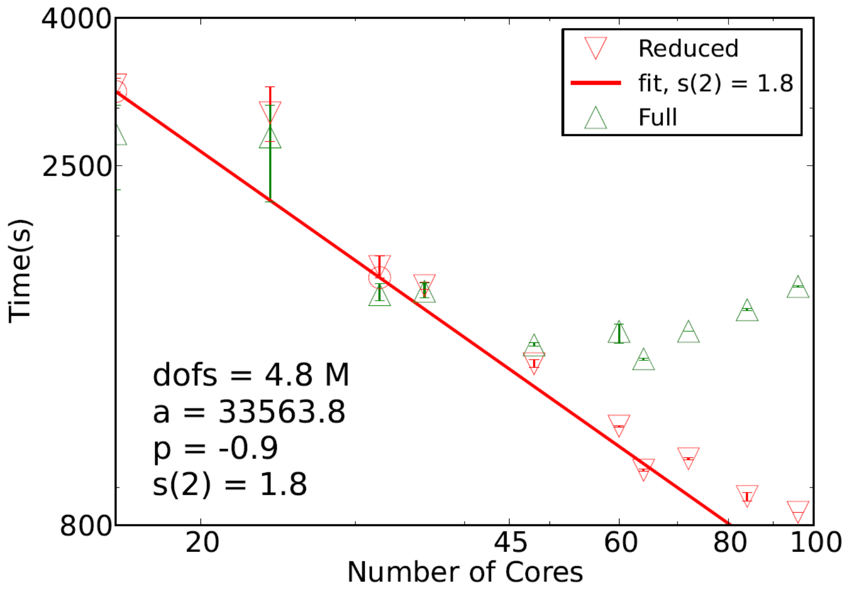
”Reduced support”

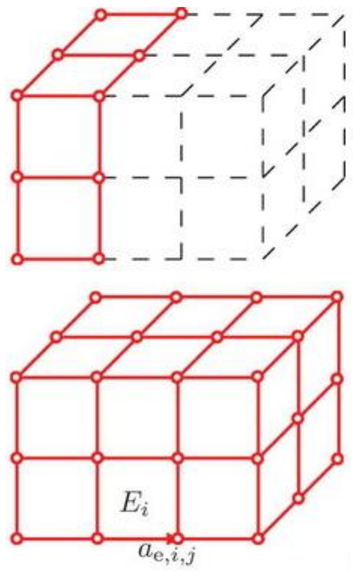
- When circuit equations are computed:
  - Matrix is of the form: $\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} a \\ i \end{bmatrix} = F$

    $$Aa + Bi = f_a$$
    $$Ca + Di = f_i$$

  - The i equations are all owned by the last process
    - Aa and Ca might be (and usually are) computed in a different process than Bi and Di => communication to and from the owner of i equations depends on the coupling between the a and i that are in different processes
- Reduced support minimizes the sizes of the coupling matrices B and C => reduced communication
  - Improved scaling with dense meshes is expected

HPC highlight: Reduced support vs full support [2]

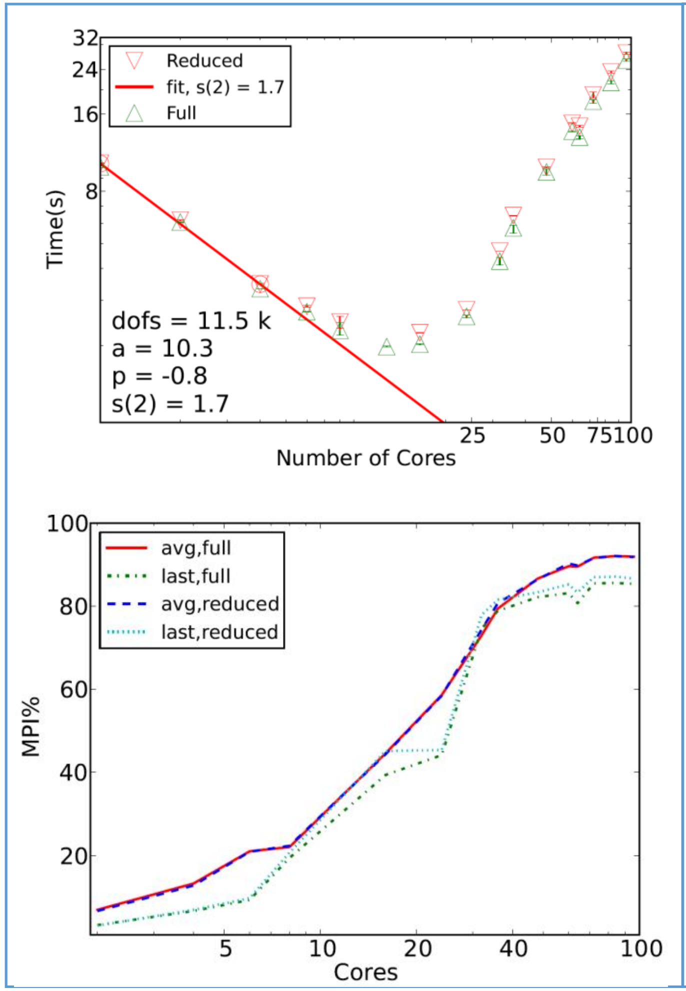




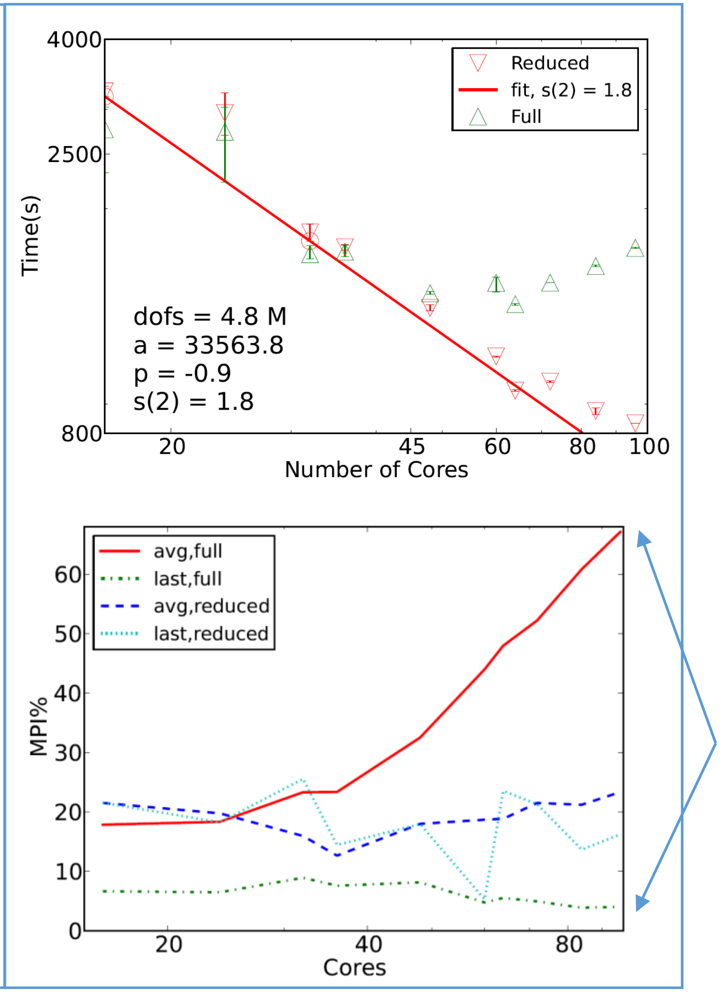
[2] E. Takala et. al. ”Using Reduced Support to enhance parallel strong scalability in 3D Finite Element Magnetic Vector Potential Formulations with Circuit Equations”, Electromagnetics 36(6), August 2016, p. 400-408

Full vs Reduced Support

Coarse Mesh



Dense Mesh



MPI% is the relative computation time used in the MPI routines (communication)

Last processor was chosen as the owner of the circuit equation => it is working full steam while the others are waiting for the communication. This is seen in the MPI%

Interested in using Elmer?

Try it!

Looking for an Elmer consultant?

- Electromagnetism
- Circuits
- Mechanics
- Heat Transfer
- Coupled physics

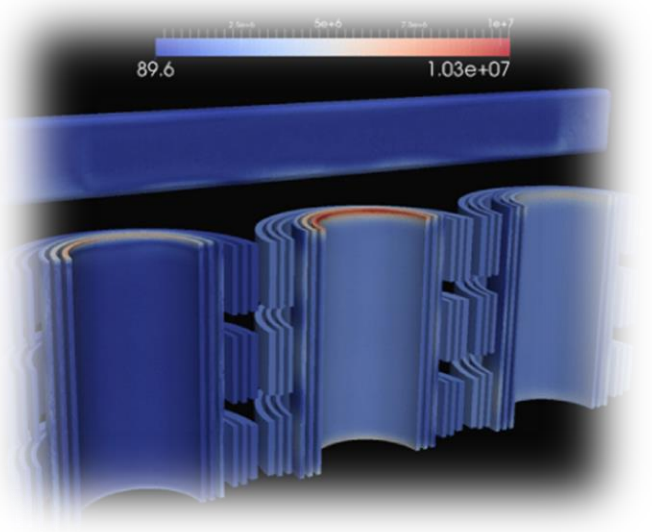
Contact me now!

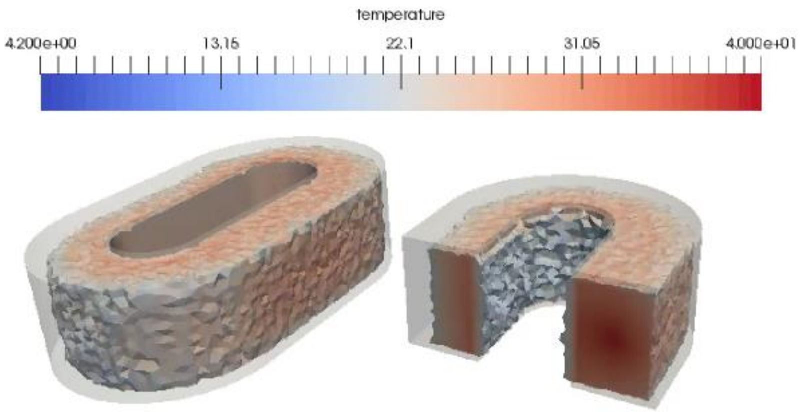
eelis.takala@gmail.com

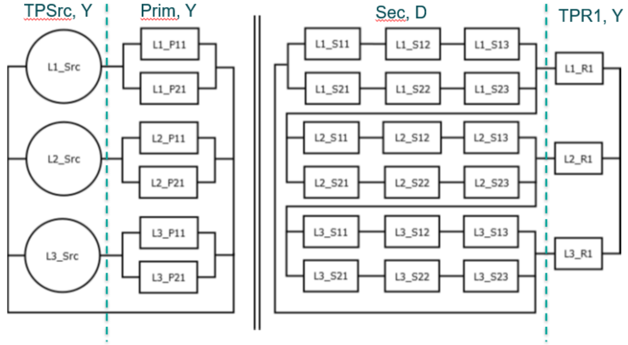
https://github.com/ettaka

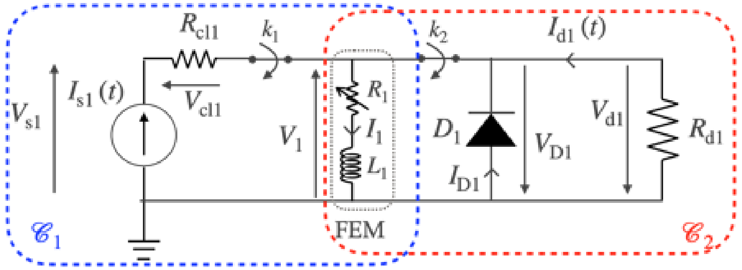
https://fi.linkedin.com/in/eelis-takala-61477554

- Title: Elmer-Circuits
- Introduction to elmerfem module for circuit equations and FE model coupling: Stranded, massive and foil windings; homogenization; parallel simulations (MPI). Basic usage and the role in 3-phase power transformer simulations is presented. Exciting new developments in superconducting magnet quench protection circuits are explained.


89.6
1.03e+07


temperature
4.200e+00
13.15
22.1
31.05
4.000e+01


TPSrc, Y
Prim, Y
Sec, D
TPR1, Y
L1_Src
L2_Src
L3_Src
L1_P11
L1_P21
L2_P11
L2_P21
L3_P11
L3_P21
L1_S11
L1_S12
L1_S13
L1_S21
L1_S22
L1_S23
L2_S11
L2_S12
L2_S13
L2_S21
L2_S22
L2_S23
L3_S11
L3_S12
L3_S13
L3_S21
L3_S22
L3_S23
L1_R1
L2_R1
L3_R1


R_{cl1}
k_1
k_2
$I_{d1}(t)$
V_{s1}
$I_{s1}(t)$
V_{cl1}
V_1
R_1
I_1
L_1
D_1
V_{D1}
I_{D1}
V_{d1}
R_{d1}
FEM
$\mathscr{C}_1$
$\mathscr{C}_2$