Mesh related features in Elmer

ElmerTeam
CSC – IT Center for Science, Finland

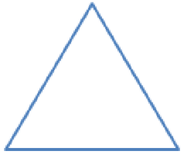
CSC, 2018

Outline

- Supported element types
    - Shapes
    - Basic functions
- Mesh generation within ElmerSolver
    - Mesh multiplication
    - Mesh extrusion
- Adaptivity – very limited
- Mesh deformation & movement
- Mesh projectors
    - Mapping between meshes
    - Mortar finite elements

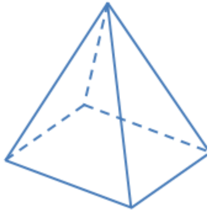
ElmerSolver – Finite element shapes

- All standard shaper of Finite Elements are supported
  - 0D: point
  - 1D: segment
  - 2D: triangles, quadrilaterals
  - 3D: tetraherdons, wedges, pyramids, hexahedrons
- Meshes may have mixed element types
- There may be also several meshes in same simulation

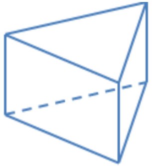


Triangle

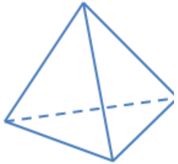
Quadrilateral



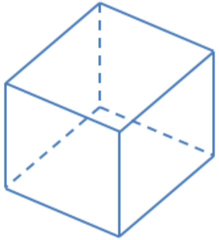
Pyramid



Prism with triangular base



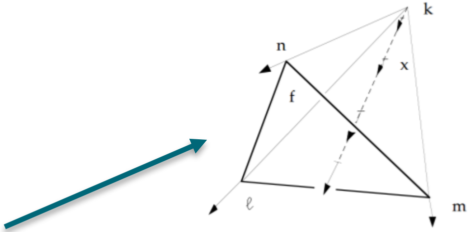
Tetrahedron

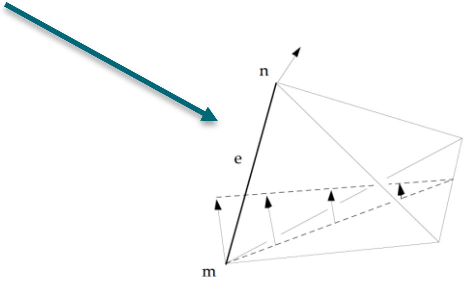


Hexahedron

ElmerSolver – basis functions

- Element families
  - Nodal (up to 2-4th degree)
  - p-elements (up to 10th degree)
  - Edge & face –elements
    - H(div) - often associated with”face” elements)
    - H(curl) - often associated with ”edge” elements)

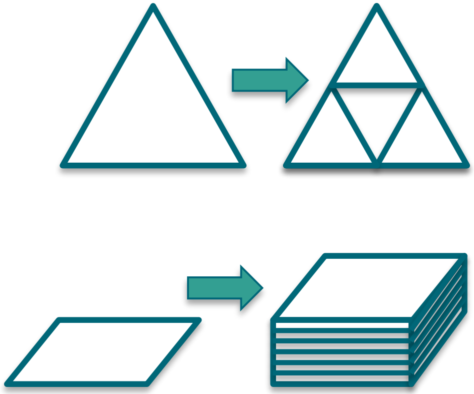




- Formulations
  - Galerkin, Discontinuous Galerkin
  - Stabilization
  - Residual free bubbles

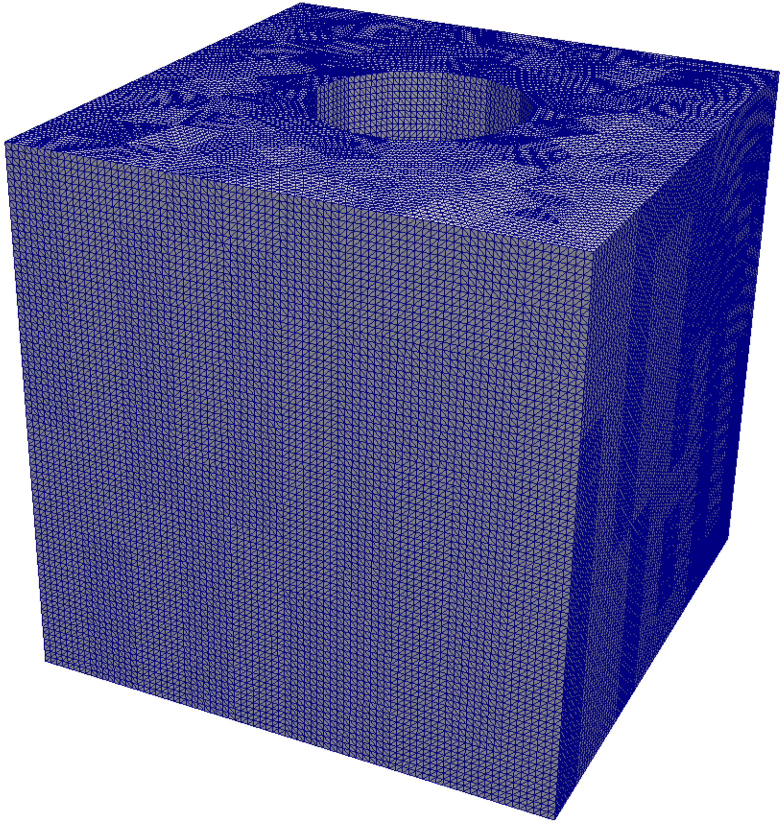
ElmerSolver – internal mesh generation

- Internal mesh division
  - *2^DIM^n* -fold problem-size
  - Known as "**Mesh Multiplication**"
  - Simple inheritance of mesh grading
- Internal mesh extrusion
  - Extruded given number of layers
- Idea is to remove bottle-necks from mesh generation
  - These can also be performed on a parallel level
- Limited by generality since the internal meshing features cannot increase the geometry description



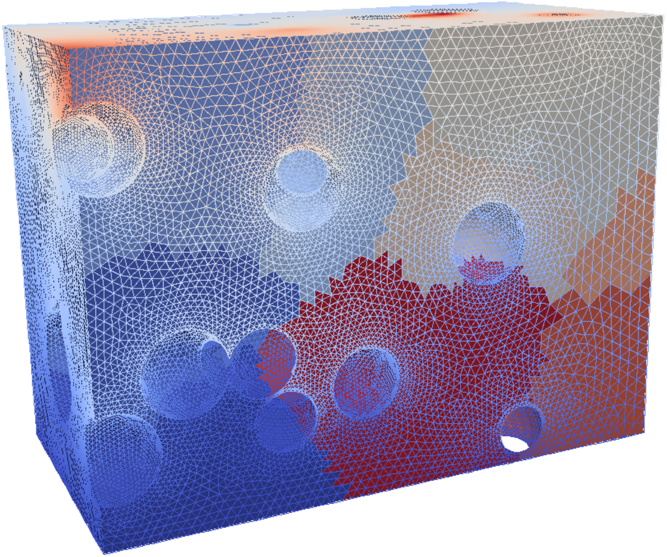
Mesh multiplication example

Mesh Levels	Number of Elements
1	7 920
2	63 360
3	506 880
4	4 055 040



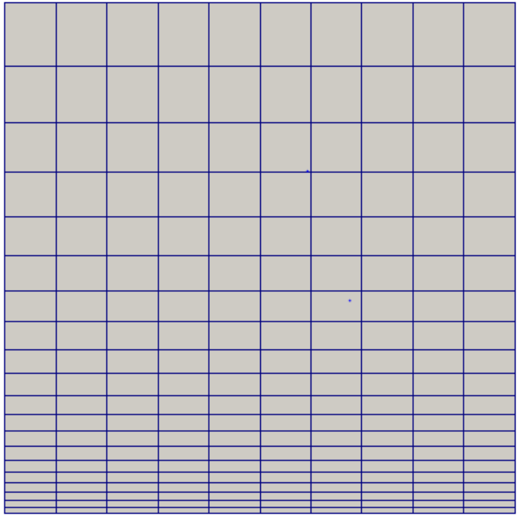
Limitations of mesh multiplication

- Standard mesh multiplication does not increase geometric accuracy
  - Polygons retain their shape
  - Mesh multiplication could be made to honor boundary shapes but this is not currently done
- Optimal mesh grading difficult to achieve
  - The coarsest mesh level does not usually have sufficient information to implement fine level grading



ElmerSolver - Internal mesh extrusion

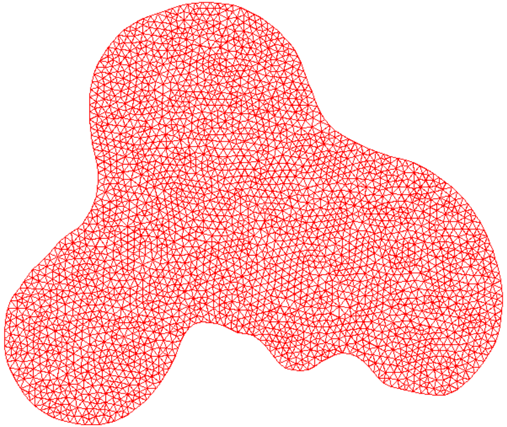
- Start from an initial 2D (1D) mesh and then extrude into 3D (2D)
  - Mesh density may be given by arbitrary function
- Implemented also for partitioned meshes
  - Extruded lines belong to the same partition by construction!
- There are many problems of practical problems where the mesh extrusion of a initial 2D mesh provides a good solution
  - One such field is glasiology where glaciers are thin, yet the 2D approach is not always sufficient in accurary



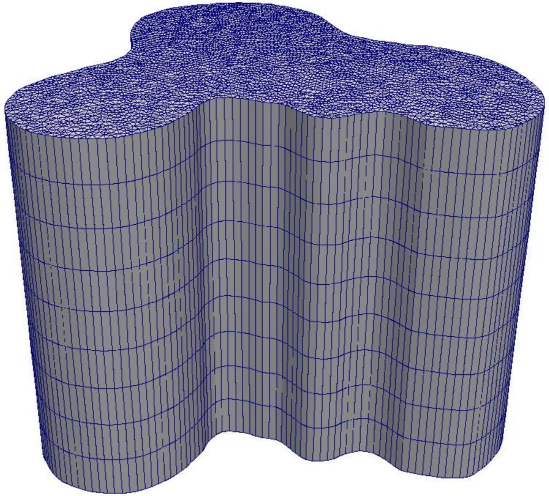
```
Extruded Mesh Levels = 21
Extruded Mesh Density =
  Variable Coordinate 1
  Real MATC "1+10*tx"
```

ElmerSolver - Internal extrusion example

Design Alvar
Aalto, 1936



2D mesh by Gmsh



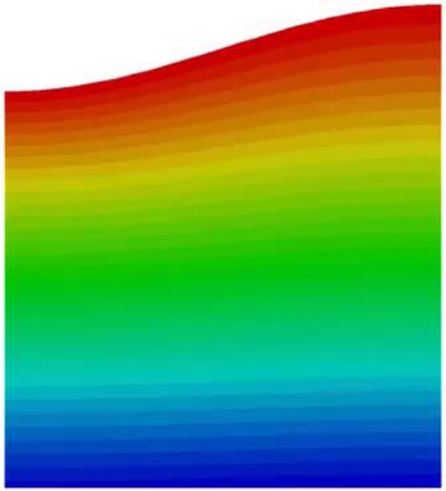
3D internally extruded mesh

Summary: Alternatives for increasing mesh resolution

- Use of higher order nodal elements
  - Elmer supports 2nd to 4th order nodal elements
  - Unfortunately not all preprocessing steps are equally well supported for higher order elements
    - E.g. Netgen output supported only for linear elements
- Use of hierarhical p-element basis functions
  - Support up to 10th degree polynomials
  - In practice **Element = p:2**, or p:3
  - Not supported in all Solvers
- Mesh multiplication
  - Subdivision of elements by splitting

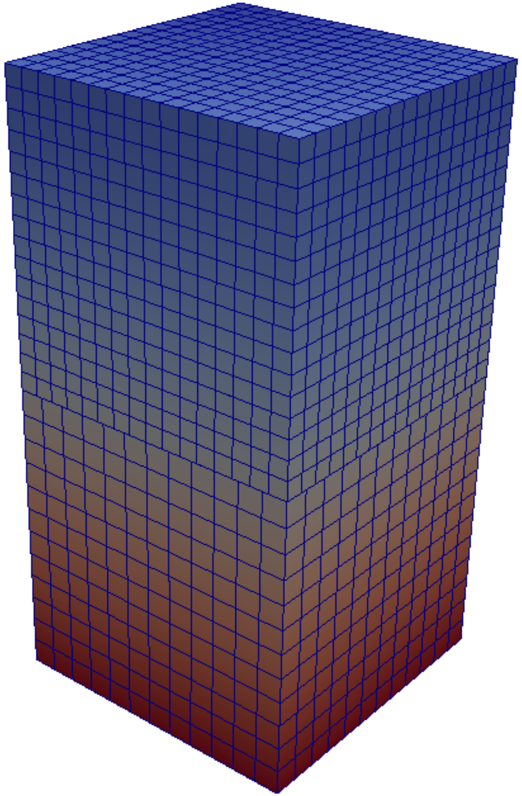
ElmerSolver – Mesh deformation

- Meshes may be internally deformed
- **`MeshUpdate`** solver uses linear elasticity equation to deform the mesh
- **`RigidMeshMapper`** uses rigid deformations and their smooth transitions to deform the mesh
- Deforming meshes have number of uses
  - Deforming structures in multiphysics simultion
    - E.g. fluid-structure interaction, ALE
  - Rotating & sliding structures
  - Geometry optimization
    - Mesh topology remains unchanged



Mapping & Projectors

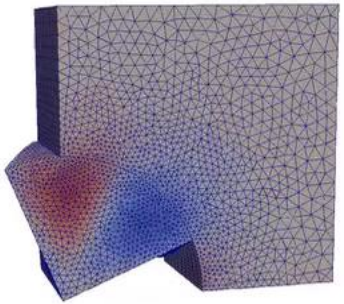
- Ensuring continuity between conforming and nonconforming meshes
  - For boundary and bulk meshes
- On-the-fly interpolation (no matrix created)
  - Mapping of finite element data
    - from mesh to mesh
    - From boundary to boundary
- Creation of interpolation and projection matrices
  - Strong continuity, interpolation: $x_l = Px_r$
  - Weak continuity, Mortar projector: $Qx_l - Px_r = 0$



Tie contact in linear elasticity using mortar finite elements

Example: Mesh utilities applied to rotational problems

- Rigid body movement may be used to implement rotation
- One of several contact pairs are used to define mortar projectors that ensure continuity of soluton
- Most important application area has been the simulation of electrical machines



Concluding remarks on internal meshing features

- Internal meshing features can be used to resolve number of challenges related to meshes
    - Accuracy
    - I/O bottle-necks
    - Continuity requirements
    - Multiphysics coupling
    - Deforming or moving computational domains